Mechanism and Machine Theory 112 (2017) 123–144

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Research paper

Dynamics of low speed geared shaft systems mounted on rigid bearings

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ARTICLE INFO

Article history:
Received 30 July 2016
Revised 12 January 2017
Accepted 12 February 2017

Keywords:
Linear dynamics
Geared shaft
Extended operator
Assumed modes method

ABSTRACT

Analytical model for the linear vibration analysis of a pair of coupled spur geared shaft system is developed. The model is a hybrid discrete-continuous one in which the gears are modeled as rigid disks mounted on the elastic shafts having transverse as well as torsional flexibilities and supported by rigid bearings. The non-dimensional governing equations along with the natural boundary conditions are developed using the Hamilton's principle. An extended operator formulation is employed to simplify the system to a compact analytical form and demonstrate that the system is self-adjoint. The natural frequencies and vibration modes are determined by discretizing the system using the assumed modes method, where the critical feature is the use of Lagrange multipliers to enforce the matching boundary conditions at the disk-shaft interfaces. The sensitivities of the natural frequencies to various system parameters are examined and correlated with the modal energy distribution. Response due to the loaded static transmission error at the gear mesh is presented.

1. Nomenclature

Subscripts 1 and 2 denote the quantities belonging to the first and the second shafts, respectively. Overbars indicate dimensional quantities.

Otherwise, these are non-dimensional.

t	Time
Ω_1, Ω_2	Rotational speeds of the shafts
ω_i	Natural frequency of the ith mode
$\bar{L}_1, \bar{L}_2$	Lengths of the shafts
a, b	Lengths of the left hand portions of the first and the second shafts
h_1, h_2	Thickness of the disks
h_{c1}, h_{c2}	Distances of the centers of mass of the disks from the left hand ends of the disks
R_1, R_2	Radii of the disks
u_1, u_2	Flexural deflections of the shafts in the uncoupled plane
v_1, v_2	Flexural deflections of the shafts in the coupled plane

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http://dx.doi.org/10.1016/j.mechmachtheory.2017.02.002

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u_{c1}, u_{c2}	Flexural deflections of the centers of mass of the disks in the uncoupled plane
v_{c1}, v_{c2}	Flexural deflections of the centers of mass of the disks in the coupled plane
θ_1, θ_2	Torsional deflections of the shafts about their respective axes
m_1, m_2	Masses of the disks
$\bar{\rho}_{s1}$, $\bar{\rho}_{s2}$	Shaft masses per unit length
K_5	Mass per unit length of the second shaft
I_{cy}, I_{cz}	Mass moments of inertia of a disk with respect to the y and z axes, respectively
K_3, K_4	Torsional moments of inertia of the shafts
I_m	Mass moment of inertia of a disk with respect to the x axis
J_1, J_2	Polar moments of inertia of shaft cross-sections
I_{sy}, I_{sz}	Area moments of inertia of a shaft cross section with respect to the y and z axes, respectively
k_m	Mesh stiffness
K_1, K_2	Torsional rigidities of the shafts
K_6	Flexural rigidity of the first shaft in the coupled plan
K_7, K_8	Flexural rigidities of the second shaft in the uncoupled and the coupled planes, respectively

2. Introduction

Rotating machineries are extensively used for power generation and transmission. Chain, belt, and gear drives are frequently used in these machineries, the purpose being connecting different branches for transmission of power, variation of speed, and change of direction of work. Extensive research has been done on the vibration induced by the gear mesh. Gears mounted on short shafts are assumed to be rigidly mounted laterally, making the gear mesh the only source of vibration and noise. The same assumption is invalid for gears mounted on long, slender shafts, falling under the class of geared rotor dynamics problems.

A number of methods are available in literature to solve the vibration analysis problems of a flexible and/or rigid shaft carrying flexible or rigid single or multiple disks. Some of the previous methods were based on lumped parameter approximation using transfer matrix methods [1,2]. Lumped parameter approach simplifies the problem but does not represent a true physical model and hence, does not accurately predict the higher natural frequencies and critical speeds of a shaft-disk system. The problem of a continuous shaft carrying a rigid disk was solved by Srinath et al. [3] for the non-rotating case and Eshleman et al. [4] for the rotating case. Characteristic equation method was used to solve system eigenvalues and effect of the thickness of the disk was ignored in both the cases. Chivens et al. [5] determined the influence of disk flexibility on the natural frequency of bending and critical speeds of a rotating disk-shaft system. Nataraj [6] developed a mathematical model to obtain the interaction between the torsional and flexural deflections of a uniform shaft rotating at a constant speed. Coupled vibration among shaft torsion and blade bending of a multi-disk rotor system with grouped blades was analytically investigated by Chiu et al. [7]. Shaft bending was ignored in the formulation. Shen et al. [8] examined the coupled vibration of multiple flexible disks and the rocking motion of the attached short, rotating, rigid shaft. Lee et al. [9] used assumed modes method to find the effect of disk flexibility on the vibration modes of a rotating flexible shaft carrying multiple flexible disks. Hili et al. [10] used finite element method to solve the eigenvalue problem of a spinning flexible disk-shaft system. Natural frequency comparison between systems with and without disk flexibility was performed to show that both disk flexibilty and shaft boundary conditions affect the system behavior.

Study of geared spindle system is more recent. This system is usually represented as two disk-spindle systems coupled through a gear mesh. In most of the cases, the disks are considered to be rigid, and the shafts are subject to torsional vibrations. If the shafts are compliant enough, the lateral vibrations due to flexure may not be neglected. For a general rotor system with length-to-diameter ratio less than 100, coupling between the flexural and torsional vibrations may be neglected [4]. However, for a geared disk-spindle system, due to the gear meshing effect, this coupling phenomenon may arise for a length-to-diameter ratio well below 100. Analytical solution for the coupled flexural, torsional, and mesh vibration characteristics of a geared rotor system is rare. In one of the pioneering works, Iida et al. [11] obtained the natural frequency and vibration modes for coupled torsional-flexural vibrations of geared rotors. However, in their model, one of the shafts was assumed to be rigid, and the response to geometrical eccentricity and mass unbalance of the three degree of freedom model was determined. Effect of gear mesh was ignored. Later, Iida et al. [12–14] applied this model to a gear train. David et al. [15] showed the importance of including dynamic coupling of system elements to produce tractable solution of a torsional-axial-lateral coupled geared systems. Effect of shaft inertia in gear dynamics for a geared shaft system using linear continuous torsional shaft model was first studied by Sener et al. [16]. Ozguven [17] conducted dynamic analysis of a six-degree-of-freedom non-linear model consisting of a spur gear pair with time varying mesh stiffness and backlash. Transverse vibration of the gears perpendicular to the direction of the line of action was neglected. Shaft dynamics was neglected while dynamics of the bearings were considered in his model. Neriya et al. [18] and Kahraman et al. [19] used finite element methods to study the modes and the dynamic response of the coupled torsional-flexural model of geared rotors. Using transfer matrix method, Choi et al. [20] showed that the lateral motion along the direction of the gear mesh might induce the coupled lateral torsional mode for a geared rotor-bearing system. Lee et al. [21] examined the coupled flexural and torsional vibration characteristics of a geared rotor bearing system by finite element method. They systematically showed that some modes yield coupled phenomena when the gear mesh stiffness is high. While finite element methods are straight-