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Research paper

A general and efficient multiple segment method for kinetostatic analysis of planar compliant mechanisms

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ABSTRACT

In the past decades, numerous mechanics models and mathematical formulations have been developed for kinetostatic analysis of compliant mechanisms. However, it is rather tedious and error-prone to derive analysis equations based on these models. In this work, we present a general kinetostatic analysis framework for planar compliant mechanisms in which 2D beams can be represented by multiple segments of three commonly used models: beam-constraint-model (BCM), linear Euler–Bernoulli beam and pseudo-rigid-body models (PRBM). The framework is developed such that any beam model with a closed-form energy equation can be integrated without the deep understanding of the proposed scheme. The static equilibrium equations are automatically derived based on kinematic vector loop and solved based on minimization of total potential energy. Since the PRBM only returns the tip deflection, we have developed a procedure for calculating strain energy, actual beam shape and bending stresses from the tip deflection. This framework has been implemented DAS2D, an open-source object oriented software.

1. Introduction

Compliant mechanisms [1] utilize the deflections of the flexible members to transfer or transform energy, force and motion. Complaint mechanism have several advantages over the traditional rigid-body mechanisms, such as increased performance, lower cost and easy miniaturization. Deflection of the elastic members in a compliant mechanism has been well studied over the past decades and there are a number of approaches to analyze large deflections of flexible beams in compliant mechanisms.

Pseudo-rigid-body methods [2–4] approximate the flexible beams with a series of rigid members connected with torsion or linear springs. Rigid body kinematic and kinetostatic analysis are well established and therefore, PRBM is intuitive in design and analysis of compliant mechanisms. The pseudo-rigid-body model method has a variety of application areas, ranging from designing carbon nanotubes [3], DNA origami mechanisms [5] or safe robot arms [6]. The parameters of the PRBM method can be load independent [7–9] and axial expansion of the flexible beam can be captured accurately by employing linear springs [10–12] in the PRBM.

The differential equations for the large deflection Euler–Bernoulli beam and the Timoshenko beam [13] can be solved numerically and also, EIS (*e*lliptic integral solution) [14–16] method employs elliptic integrals [17] to iteratively solve the

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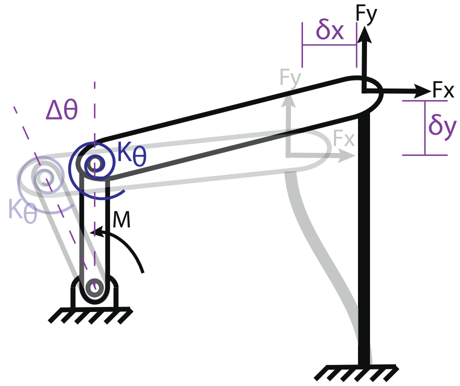

Fig. 1. An example compliant mechanism consisting of a flexible link and a torsion spring.

Euler–Bernoulli beam equation. Similarly, various semi-analytical Adomian decomposition methods [18,19] have been proposed to solve large deflection equations of cantilever beams.

Beam-constraint-model (BCM) [20–22] was developed in order to accurately capture intermediate nonlinear deformations of a cantilever beam within ten percent slope change. BCM method have parametric and closed form equations for calculating deflection, strain energy and bending stress of a beam.

A number of multiple segment models exist for large deflection of flexible members. The chain algorithm [23] discretize the beam into many small deflection beam elements fixed to each other at the ends. Static analysis is performed by incrementally increasing the end loads. Chained BCM method [24] consists of multiple BCM segments cantilevered at the free ends while greatly increasing the accuracy of the analysis with respect to using a single BCM beam. The final deflection is calculated by iteratively solving the individual BCM equations with the geometric constraint and load equilibrium equations. Finite element method (FEM) [25] is one of the most commonly used methods in analyzing compliant mechanisms. Various commercial FEM based software is available for analysis of compliant mechanisms.

Kinetostatic analysis of compliant mechanisms can be performed via minimization of total potential energy using PRBM [26,27]. This paper proposes an unified method for integrating any beam deflection model with a closed form energy equation into the minimization framework. Since optimization itself is an iterative process, it is important that energy of a beam can be calculated directly in order not to have second level iterations. BCM and linear Euler–Bernoulli beam [28] are given as two examples that fit well into the proposed scheme.

The paper is organized as follows. Section 2 presents the basis of the energy minimization method for compliant mechanisms. Newly developed multi-segment framework is introduced in Section 3 with the derivations of the energy equations of the Beam-Constraint-Model and linear Euler–Bernoulli beam in the form of the proposed framework. The next section proposes a Euler–Bernoulli based method for obtaining beam shape and stress data from PRBMs. DAS2D, an open-source software based on the proposed framework, is presented in Section 5. Four case studies ranging from a cantilever beam to a fully compliant inverter mechanism are provided in the last section.

2. The potential energy minimization architecture for kinetostatic analysis

There are severals ways to analyze elastic deformation of compliant mechanisms ranging from linear direct stiffness [29] approach to powerful finite element analysis [25]. Analytical methods such as the virtual work [30,31], exists but they are usually inconvenient to calculate even numerically since resulting equations are pretty complicated. For these methods, it is quite cumbersome to incorporate a different beam model since none of the approaches have the flexibility to be easily adapted without constructing the equilibrium equations.

Energy minimization based methods [26,27] provide a simple but powerful approach for automating the kinetostatic analysis of compliant mechanisms without constructing static force equilibrium equations. Alternatively, the total potential energy of the system constrained by kinematic equations can be minimized and for a general compliant mechanism (Fig. 1), the energy minimization problem can be stated as:

$$\min_{\psi} \; f = U(\psi_1, \psi_2, \ldots, \psi_n) + V(\psi_1, \psi_2, \ldots, \psi_n)$$

$$\text{subject to } g = \sum_{i=1}^{c} \vec{Z}_i(\psi_1, \psi_2, \ldots, \psi_n) = 0 \tag{1}$$

where ψ is the list of optimization variables, U is the strain energy stored in the flexible beams or in the energy storage elements, V is the negative of the work done by external loads and g is the kinematic constraint equations (vector loop equations).