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Research paper

Applying screw theory for summing sets of constraints in geometric tolerancing

Santiago Arroyave-Tobón*, Denis Teissandier, Vincent Delos

Univ. Bordeaux, I2M, UMR 5295, F-33400 Talence, France

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ABSTRACT

In tolerance analysis, approaches based on sets of constraints (also called convex hull techniques) are able to study simultaneously all the possible extreme configurations of a mechanism when simulating manufacturing defects in its components. The accumulation of these defects can be calculated by summing and intersecting 6-dimensional sets of constraints, i.e. polyhedra. These approaches tend to be time-consuming, however, because of the complexity resulting from manipulating sets in $\mathbb{R}^6$. In this paper, polyhedra are decomposed into a bounded set (a polytope) and an unbounded set (a set of straight lines). The unbounded part of the polyhedra is characterized by the degrees of freedom of the toleranced feature or the joint. Therefore, the decomposition can be performed based on a kinematic analysis of the studied assembly using screw systems. The proposed decomposition is presented for the most common features used in geometric tolerancing. The idea behind this strategy is, instead of summing polyhedra in $\mathbb{R}^6$, to sum only their underlying polytopes by isolating the unbounded part of the operands. A slider-crank mechanism is used to show the gain in computational time of the proposed method in comparison with the strategy based on complete 6-dimensional sets of constraints.

1. Introduction

Tolerance analysis consists in studying the impact of manufacturing defects on product behavior. The objective is to determine if the accumulation of defects in the different parts allows the ideal functioning of the product. One way to perform tolerance analysis of three-dimensional tolerance chains is by means of sets of constraints. Using these approaches, it is possible to simulate simultaneously all possible displacements (3 rotations and 3 translations) of a toleranced feature inside its tolerance zone or the relative movements of two features of two parts mated by a contact restriction [1]. They allow, additionally, the treatment of overconstrained (hyperstatic) mechanisms. Some models consider quadratic constraints (such as Domains [2] and T-Maps [3]), while others are based on linear constraints (such as Polytopes [4]) by considering manufacturing defects as small displacements [5]. A comparison of these methods is presented in [6]. As all possible configurations of each toleranced feature must be considered, the cumulative defect limits on parts in a serial configuration can be calculated by means of the Minkowski sum of sets of geometric constraints [1,7]. In the case of parts with parallel contacts, all the tolerance zones must be satisfied and then the intersection of the respective sets of constraints is required. By means of these two operations, the stack-up of deviations in any tolerance chain can be computed [8].

* Corresponding author.
E-mail address: santiago.arroyave-tobon@u-bordeaux.fr (S. Arroyave-Tobón).

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The degrees of freedom of contact features or the degrees of invariance of geometric features imply unbounded displacements [9,10]. In these cases, the respective set of constraints defines a polyhedron in $\mathbb{R}^6$. When the toleranced feature has no degrees of freedom or invariance (as in the case of a complex surface), the set of constraints represents a polytope (a bounded polyhedron).

On the one hand, Minkowski sums of polytopes have been a research subject explored with applications in biology [11], robotics [12,13], tolerance analysis [14,15], among others. Because of their complexity, however, the computation of sums in spaces of dimension greater than 3 is still a challenging matter. Mansuy et al. [16] propose a method for calculating separately only the most disadvantageous vertices with respect to a functional polytope. Even if this method avoids the computation of Minkowski sums, the set of computed vertices is only representative of a given functional condition. In addition, the authors just consider the case of tolerance chains made up of features of the same invariance class and in a particular relative position (i.e. a set of parallel planes). In these cases, the displacements linked to degrees of freedom or invariance can be trivially isolated from the computations.

On the other hand, for the Minkowski sums of polyhedra, just a few studies have been proposed. Homri et al. [17] suggested turning polyhedra into polytopes by introducing virtual boundaries, called cap half-spaces. This strategy has to cope with the multiplication of cap half-spaces during the computation of Minkowski sums. As a consequence, the time taken for computing cap facets (facets associated with cap half-spaces) is in general far greater than that needed for computing facets representing real limits of bounded displacements. Fukuda [18] presented an algorithm to compute Minkowski sums of polytopes, mentioning the possibility of applying the same procedure for the case of polyhedra with at least one vertex (pointed polyhedra) by treating infinite rays as points at 'infinity'. However, due to the degrees of freedom (or invariance), the polyhedra manipulated in tolerancing usually do not have vertices. Each degree of freedom (or invariance) implies a sweeping operation of a polytope along a straight line, placing the vertices at infinity.

The set of displacements required to completely define the relative position between two features in a tolerance chain can be identified by a kinematic analysis. At a later stage, the objective of tolerance analysis is to determine the limits of these displacements. For this reason it is common to find tolerance analysis techniques in the literature involving kinematic analysis [19–22]. However, it has been done just for the case of parametric techniques (which are not able to treat assemblies with redundantly suppressed degrees of freedom). The advantages of applying kinematic analysis in the case of techniques based on sets of constraints remain unexplored, and this is actually the objective of this paper.

One possible way to model rigid body motions is by means of the theory of Euclidean group displacements. This theory states that in the affine Euclidean space of dimension 3, the set of rigid transformations has an algebraic structure of a continuous group, more specifically it is a Lie group of dimension 6 [23]. Therefore, the union of two successive displacements is also a displacement [24]. Fanghella [25] presented an exhaustive list of products of subgroups. The author produced this classification by analyzing systematically all possible couples of subgroups and their geometric relations (coincidence, parallelism, intersection, etc.). Hervé [26] presented a more general procedure by using the exponential maps of the Lie groups enumerating 12 possible kinds of displacement subgroups. However, the 12 displacement subgroups cannot explain all rigid motions in space. Under most conditions, a rigid motion is simply a displacement manifold included in the displacement group but not a subgroup itself [27].

In a similar way, using the theory of screws [28] it is possible to perform spatial kinematic analysis. By computing systematically the union and intersection of screws the mobility condition of any two surfaces of a mechanical system can be determined [29–31]. Open serial and closed chains can be analyzed even if the former present redundantly suppressed degrees of freedom. For these purposes, the classification of screw systems plays an important role. Rico Martinez and Duffy [32,33] proposed a classification based on the reciprocal basis of screw systems. A general approach for mechanisms analysis and synthesis based on this reciprocity is proposed by Dai and Jones [34].

This paper proposes the use of the theory of screws for modeling the mobility conditions of an assembly during tolerance simulation with sets of geometric constraints. It enables us to simplify the sets of constraints, decomposing them into the sum of a polytope (a bounded set) and a set of straight lines (an unbounded set). When the operand sets are decomposed, their sum can be calculated by dealing with the underlying polytopes. This implies a reduction in operand complexity and consequently a reduction in the computational time. In this work, the advantages of using the proposed model (decomposed polyhedra) are explored only in the case of sums, i.e. when the tolerance chain is made up of serial contacts.

The next section of the paper introduces the way to sum sets of constraints based on polytopes and cap half-spaces. In Section 3 the new strategy based on kinematic analysis is presented. Next, in Section 4, a case study is solved using the proposed method and the results are compared with those obtained by the method based on cap half-spaces. Finally, some conclusions and perspectives are discussed in Section 5.

2. Tolerance analysis with polytopes of $\mathbb{R}^6$

In this section, the tolerance analysis method based on the operation with polytopes of $\mathbb{R}^6$ is presented with its mathematical support and illustrated with an example. For further details the reader can refer to [17].

Before going into the details of polyhedra and polytopes in the field of geometric tolerancing, let's present some definitions required for this paper. They are taken from [35].

Definition 2.1. An $\mathcal{H}$-polyhedron is an intersection of many finitely closed half-spaces in some $\mathbb{R}^n$.