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Research paper

Optimum design of multi-degree-of-freedom closed-loop mechanisms and parallel manipulators for a prescribed workspace using Monte Carlo method

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ABSTRACT

In this work, we use the Monte Carlo method in conjunction with gradient based optimization algorithms to optimally design multi-degree-of-freedom parallel manipulators and closed-loop mechanisms. The design procedure takes into account practical constraints such as joint limits and guarantees well-conditioning of the desired workspace. As a first step, an appropriate bounding box representing the wanted workspace is obtained by using the Monte Carlo method and then the geometrical dimensions of the manipulator are obtained through a gradient based optimization method by accounting for the joint and other constraints. The computational advantages of the Monte Carlo technique over other search based methods in evaluating the objective function for the optimization problem is illustrated. The constraint Lagrange multipliers are obtained and sensitivity of the workspace dimensions to the constraints on joint limits and conditioning have been demonstrated. The approach is illustrated with the design of a two-degree-of-freedom planar 5-bar closed-loop mechanism and a spatial, six-degree-of-freedom Stewart platform manipulator.

1. Introduction

Parallel manipulators are widely used for a variety of tasks where increased accuracy and large load carrying capacities are required. The most well-known spatial in-parallel manipulator, the Stewart platform has been used extensively for tire testing, aircraft simulators, machine tools and many other applications. Other platform type in-parallel manipulators, with three degrees-of-freedom such as the 3-UPU and 3-RPS manipulators have been proposed as a parallel wrist [1] for orienting an object and for tracking the sun for concentrated solar plants [2]. Hybrid parallel manipulators have been proposed as a model of multi-fingered hands (see, for example, the Stanford-JPL hand by Salisbury and Craig [3] and the Utah-MIT hand by Jacobsen et al. [4] and the 3 fingered hand by Borras and Dollar [5]). Planar multi-degree-of-freedom, closed-loop mechanisms such as the 3-RRR or a 5-bar mechanism have been used for precision manipulation in a plane, haptic devices (see Phantom range of haptic devices by Sensable [6] etc.). In most of these instances, the stress has been to obtain the solutions to the direct and inverse kinematics problems (see, for example, the pioneering works by Wen and Liang [7] and Raghavan and Roth [8] and the references contained therein), perform singularity analysis (see, for example, Bandyopadhyay

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Nomenclature and list of symbols

${}^{B}_{A}[R]$	Rotation matrix of frame $\{B\}$ with respect to frame $\{A\}$
$\Re$	Set of real numbers
$SO(3)$	Special orthogonal group of order 3
u_i	i^{th} Element of a vector $\mathbf{U}$
$\Pr(X)$	Probability of a random variable X
$E(X)$	Expectation of a variable X taking a probabilistic value
S_n	Average of n quantities $(x_1, x_2, \ldots, x_n)$
$V(X)$	Variance of a variable X taking a probabilistic value
$\widehat{X}$	Estimate of the quantity X
$\mathbf{J}$	Jacobian matrix
κ	Condition number of a matrix
$tr(\mathbf{A})$	Trace of a matrix $\mathbf{A}$
$\mathcal{O}$	The 'Big O' notation denoting computational complexity
$\mathcal{W}$	Workspace of a manipulator
S_i, U_i	Centers of spherical and universal joints, respectively

and Ghosal [9–11] and the references contained therein), derive and numerically solve the dynamic equations of motion (see, for example, the comprehensive review by Dasgupta and Mruthyunjaya [12] and the references contained therein) and for control (see, for example, the works by Hatip and Ozgoren [13], Narasimhan [14], Wang et al. [15], Wen et al. [7] etc. and the references contained therein). Unlike the extensively studied planar four-bar and other one-degree-of-freedom planar mechanisms, there is relatively less literature on the design of parallel manipulators and multi-degree-of-freedom planar or spatial closed-loop mechanisms for a given set of objectives. In this work, we focus on the problem of optimal dimensional synthesis of a parallel manipulator or a closed-loop mechanism for a specified workspace, subject to joint rotation limits and obtaining a well-conditioned workspace. In the review paper on Stewart platform manipulators [16], the authors have also recognized the above as an open area of research. Optimization of parallel manipulators and closed-loop mechanisms in terms of dimensional synthesis for the largest specified workspace and (or) highest end-effector accuracy is a continuing area of research. In the following we present a brief summary of the current state-of-the-art in this topic.

In a body of literature (see, for example, the work by Boudreau and Gosselin [17] and the book by Davidor [18] and the references contained therein), the authors have recognized the non-convex nature and difficulties in optimizing a parallel manipulator for desired characteristics and have thus not suggested the use of gradient based optimization, involving closed form kinematic equations of parallel manipulators. Genetic algorithm or other evolutionary algorithms have been chosen frequently by researchers (see, for example, the work by Grefenstette [19] and the references contained therein) for optimization problems including but not limited to dimensional synthesis. A different approach to the optimization problem (see, for example, Masory and Wang [20] and Tsai and Soni [21]) is by evaluating and maximizing the boundary curves of the feasible workspace of the manipulator at a particular plane with one of the Cartesian variables as fixed. Pittens and Podhorodeski [22] and Han et al. [23] have used gradient based optimization to obtain the dimensions of a manipulator for highest accuracy by reducing the condition number over the feasible workspace so that the accuracy of the manipulator is good everywhere in the workspace of the manipulator. Gosselin and Guillot [24] have worked on the optimization problem of planar parallel manipulators in Cartesian space. The method used by them obtains the geometric description of the intersection of the available workspace and obtained the workspace of the manipulator and subsequently minimizes the exclusion zone of the intersected workspace, thereby reaching at the optimum configuration. An avenue of research started with the pioneering works of Merlet (see e.g. [25–28] and [29] and the references contained therein) describe the use of interval analysis as a technique to determine the upper and lower bounds of a function and has proposes its use it for the optimal design of parallel mechanisms by maximizing a particular cost function. Methods based on numerical constraint programming exist, which represent and quantify non-singular workspaces of parallel manipulators. A recent work by Caro et al. [30] uses this technique in conjunction with branch and prune algorithms to compute general aspects of parallel manipulators like non-singular self-collision free workspaces. Borras and Dollar [5] have considered two versions of the same parallel manipulator – one as an under-actuated (or hybrid parallel) manipulator and another as a fully actuated version and have generated optimum dimensions of both for the maximal precision workspaces. In the work they have also computed the actual number of configurations (reported to be of the order of 10^7 for the worst case) to be searched through and have suggested random search technique to quickly go through the search space. Lou et al. ([31–33]) have used the controlled random search (CRS) method to optimize robots for regular workspaces with good dexterity. The works by Stamper et al. [34], Tsai and Joshi [35] use Monte Carlo search based methods to optimize manipulators for the largest well-conditioned workspace. The objective function is generally a representative of the quantity of the actual workspace. The works by Lou et al. and Tsai et al. ([31–34]) are closest to the current work and in this paper, we extend some of the ideas presented in these works.