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Research paper

Analytical solution of spur gear mesh using linear model

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A B S T R A C T

The paper is concerned with vibration problem in a spur gearing. A one-degree-of-freedom (1 DOF) linear and periodic system including mesh stiffness and manufacturing error is considered. The parabolic function is used to describe the stiffness of tooth pair in a single pair contact. The approach using the periodic Green's function (PGF) in the form of truncated Fourier series is applied to find an analytical periodic solution in a steady state. Moreover, the method enables to find the borders of (in)stability by means of the real eigenvalues of the so called system matrix. The presented approach enables to solve the problem even in the case when periodic stiffness is implicit function of fluctuation parameter. The stability diagrams are presented and the validation of their correctness is performed by the Floquet method. The dynamic system behaviour in a steady state is also investigated and the periodic solution obtained by the presented analytical method is compared with the results given by the Runge–Kutta continuation. A very good agreements are achieved in all cases.

1. Introduction

Many different models of the dynamic behaviour of a gear system have been developed in the past few decades. The overview paper [1] gives a brief tour of models used in gear dynamics in the period from the seventies to the eighties of last century. In this period, the works [2,3] have been also published. Particular attention was paid in both studies to the steady state response. While Hsu and Cheng [2] presented explicit expression given in terms of the fundamental matrix of the homogeneous system, Benton and Seireg [3] used a computer simulation procedure based on the phase-plane method to find resonance states and response in steady-state. The number of experimental works (see, e.g., [4,5]) on the dynamic behaviour of a spur gear pair has been simultaneously published. Such studies have shown that the linear models are unable to describe the consequence of some effects (e.g. the gear backlash) occurring during the gear mesh which are not subject of the presented study. Blankenship and Kahraman [6] developed equation to describe physical behaviour observed in rotating machinery which contain time varying system parameters arising from periodically changing contact regimes due to clearances. They proposed a general solution methodology by using a multiple term of the Harmonic Balance Method (HBM). Similar scheme has been presented in [7]. In this case the incremental HBM was applied to analyze the periodic vibrations of systems with a general form of piecewise linear stiffness characteristics. Also other authors used effectively single

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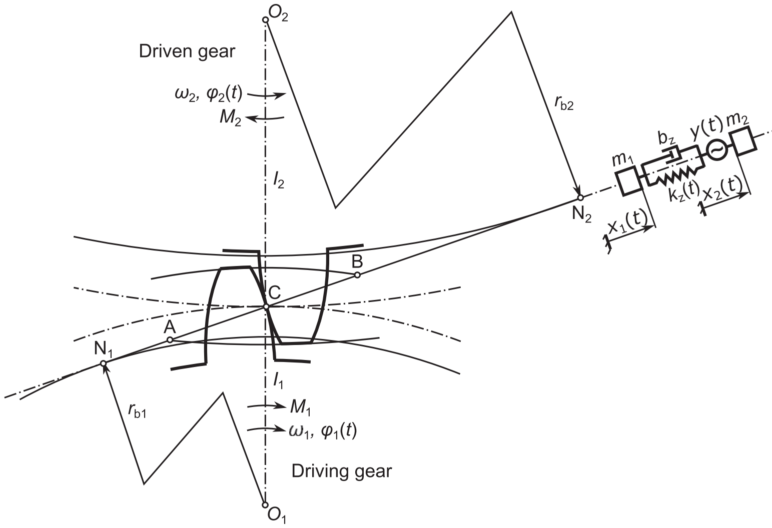

Fig. 1. Mechanical model of spur gearing.

and multiple term harmonic balance approaches to solve problems of the dynamic behaviour of a gear system. However, one limitation of HBM could be mentioned. The effects of initial conditions on the solution cannot be considered. A methodology for determining periodic steady state response of piecewise linear dynamical systems with weakly periodic coefficients was developed in [8]. Used method shares and combines characteristics of classical perturbation approaches applied to oscillators with time varying coefficients and of exact methods employed for piecewise linear systems with constant coefficients. Subsequently, Theodossiades and Natsiavas [9] extended this methodology considering more general external forcing conditions and more solution types of a gear-pair system.

In recent fifteen years, many works dealing with the dynamics of gear systems have taken into account the effects of friction and lubrication, the influence of bearing flexibility, the chaotic behaviour or the effect of cracks in the teeth, see [10–13]. Various schemes of HBM have been widely used in the solution of these problems. On the other hand, Vaishya and Singh [14] created a Coulomb friction model comprising a sliding resistance for a pair of spur gears. The forced vibration response of a system was found with the application of the Floquet theory. It should be stated at the end of this overview that a direct numerical integration is still utilized to simulate behaviour of a gear system. The Runge–Kutta and the Newmark integration methods have been most frequently applied numerical schemes in such cases, see, e.g., [15,16]. However, these schemes are more valuable as a tool for validating of analytical methods because numerical integration is not practical due to the computational expense required to obtain the steady state solutions, especially for lightly damped systems.

Although more complicated and complex mathematical models of gears are still being developed, the importance of linear models goes on. These models have the greatest significance for high speed operating gearing where the influence of gear mesh stiffness and transmission error is the most important source of internal dynamic excitation. In this context, the principal aim of this paper is to demonstrate the application possibilities of the method [17] in the investigation of the steady state response and in identifying areas of stability. This is a major advantage of this approach that the analytical solutions of systems with not only weakly periodic coefficients can be found. The analyzes are performed on the parametric 1 DOF gear system. In addition, this work provides, unlike [17], a technique for identifying the (in)stability borders in case when the fluctuation parameter is not considered as proportional to the time varying part of stiffness. Moreover, the approximation of mesh stiffness is performed using the parabolic function. This one is represented in the form of the Fourier series where coefficients are found in the analytical form. The verification of obtained results is done by the Runge–Kutta continuation and the Floquet method.

2. Gear pair model

The motion equation of investigated gear pair system is based on the typical mathematical model presented, e.g., by Theodossiades and Natsiavas [9]. The only difference is that the studied problem is solved as a linear without the influence of backlash. The dynamic model is only torsional and ignores the translational degrees of freedom. The total rotation angle of each gear corresponds to the sum of a constant angular velocity ω_j and a small variation $\theta_j(t)$ due to vibrations caused by flexibility of the tooth, i.e.

$$\varphi_j(t) = \omega_j t + \theta_j(t) \qquad \text{for} \qquad j = 1, 2\,. \tag{1}$$