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Research paper

Kinematic study of the general plane-symmetric Bricard linkage and its bifurcation variations

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In this paper, the explicit solutions to closure equations of the plane-symmetric Bricard linkage are derived and a thorough kinematic study of the general plane-symmetric Bricard linkage is conducted with DH matrix method. The derived 5*R*/4*R* linkages from this Bricard linkage are introduced. Various bifurcation cases of the plane-symmetric Bricard linkage with different geometric conditions are discussed, which include the bifurcation between two plane-symmetric Bricard linkage motion branches and the bifurcation among equivalent serial kinematic chains with revolute joints and a four-bar double-rocker linkage. Especially the plane-symmetric Bricard linkage that can bifurcate to the Bennett linkage is proposed for the first time. These findings not only offer an in-depth understanding about the kinematics of the general plane-symmetric Bricard linkage, but also bridge two overconstrained linkage groups, i.e., the Bennett-based linkages and Bricard-related ones, to reveal their intrinsic relationship.

1. Introduction

The family of 6*R* overconstrained linkages was proposed by Bricard consists of three deformable octahedral cases [1] and three spatial linkage cases [2] whose mobility is due to the symmetric property. Among them, the plane-symmetric Bricard linkage has been extensively studied. First of all, implicit solutions to the closure equation of six Bricard linkages were derived by Baker with DH loop-closure matrix method [3]. Phillips reviewed the Bricard linkages and introduced their relationship with other overconstrained linkages [4]. Baker analysed the general plane-symmetric six-screw linkage including the plane-symmetric Bricard linkage with the reciprocal screw system approach [5]. The movability of the plane-symmetric Bricard linkage was investigated by Li and Schicho based on the theory of bonds [6]. Deng et al. presented a geometric approach for design and synthesis of single loop mechanisms including the plane-symmetric Bricard linkage [7]. They also proposed a virtual chain approach for the mobility analysis of multi-loop deployable mechanisms with plane-symmetric Bricard linkage as basic element [8]. Kong conducted type synthesis of single-loop overconstrained 6*R* spatial mechanisms for circular translation in which the plane-symmetric Bricard linkage is taken as an example [9]. Even though various synthesis methods have been used to study the plane-symmetric Bricard linkage, there is no progress on the explicit solution

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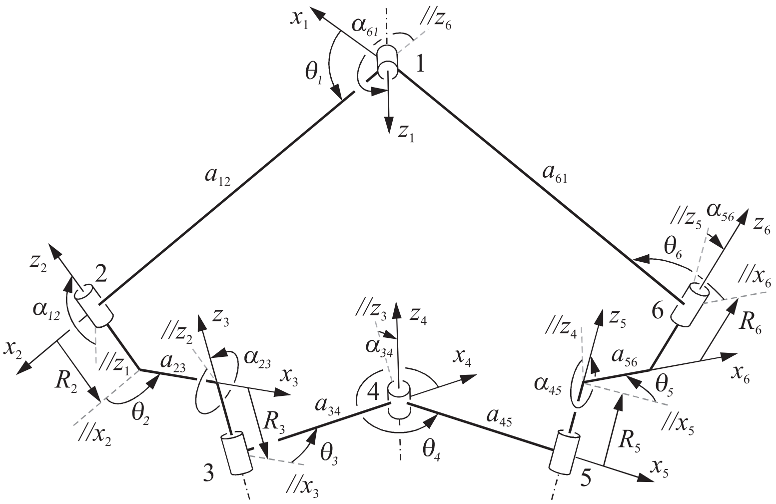

Fig. 1. D-H parameters of the plane-symmetric Bricard linkage.

of closure equations after Baker's implicit ones. The detailed kinematic behaviours of this linkage can be revealed only with the explicit solutions. Therefore, we set the target to obtain them by overcoming the complicated trigonometric functions.

Recent research applies the plane-symmetric Bricard linkage to the design of deployable structures. For example, Chen et al. proposed a threefold-symmetric Bricard linkage which is a special case of the plane-symmetric one to fold the triangular or hexagonal structures [10]. Viquerat et al. design a rectangular ring which can be folded into a compact bundle. Kinematically this is an alternative form of the plane-symmetric Bricard linkage [11]. A number of such retractable rectangular rings can form a family of large deployable mechanisms by synchronising the motion of all linkages [12].

Because of the symmetry property, the plane-symmetric Bricard 6*R* linkage tends to have complicated bifurcation behaviours, which should be avoided in the application of deployable structures, but could be made use of in the design of reconfigurable mechanisms. The kinematics and bifurcation behaviour of a special line- and plane-symmetric Bricard linkage was analysed using the SVD numerical method by Chen and Chai [13]. Zhang and Dai analysed motion branch variations of the line- and plane-symmetric Bricard linkage based on reciprocal screw systems [14]. Recent work on thick-panel origami shows 6-crease origami pattern can be replaced with the plane-symmetric Bricard linkage in the thick-panel model [15]. Moreover, the kinematic model of the thick-panel waterbomb origami is the assembly of two types of plane-symmetric Bricard linkages with bifurcation under certain geometric conditions [16]. Hence, the current bifurcation analysis of the plane-symmetric Bricard linkage only focuses on special cases. Therefore, this paper also aims to setup the general geometric condition of the bifurcation for the plane-symmetric Bricard linkage.

The layout of this paper is as follows. The explicit solutions to closure equations of the general plane-symmetric Bricard linkage are derived and the comparison between kinematics variations of different plane-symmetric Bricard linkages based on these solutions are discussed in Section 2. Section 3 introduces the derived 5*R*/4*R* linkages from the general case and their corresponding geometric conditions. Section 4 addresses the bifurcation between the plane-symmetric Bricard linkage and the Bennett linkage. Section 5 discusses other various bifurcation cases of the plane-symmetric Bricard linkage under different geometric conditions. Final conclusions are drawn in Section 6.

2. The explicit solutions to closure equations and kinematic properties of the general plane-symmetric Bricard linkage

The geometrical parameters of the general plane-symmetric Bricard linkage are shown in Fig. 1 with the conditions that

$$\begin{aligned} &a_{12} = a_{61} = a,\ a_{23} = a_{56} = b,\ a_{34} = a_{45} = c,\\ &\alpha_{12} = 2\pi - \alpha_{61} = \alpha,\ \alpha_{23} = 2\pi - \alpha_{56} = \beta,\ \alpha_{34} = 2\pi - \alpha_{45} = \gamma,\\ &R_1 = R_4 = 0,\ R_6 = -R_2,\ R_5 = -R_3. \end{aligned} \tag{1}$$

The setup of coordinate frames is in accordance with the Denavit and Hartenberg's convention [17], where z_i is along the revolute axis of joint *i;* x_i is the common normal direction pointing from z_{i-1} to z_i; $a_{i(i+1)}$ is the normal distance between z_i and z_{i+1}(also known as the length of link $i(i+1)$); $\alpha_{i(i+1)}$ is the angle of rotation from z_i to z_{i+1} about axis x_i (also known as the twist of link $i(i+1)$); R_i is the normal distance between x_i and x_{i+1} (also known as the offset of joint *i*); and θ_i is the angle of rotation from x_i and x_{i+1} about axis z_i (also known as the kinematic variable of joint *i*). Here, *a*, *b*, *c*, α, β, γ, R_2 and R_3 are taken as the geometrical parameters of the plane-symmetric Bricard linkage.