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Research paper

A geometric algebra approach to determine motion/constraint, mobility and singularity of parallel mechanism

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ABSTRACT

The crucial procedure of mobility and singularity identification of parallel mechanisms is widely recognized as how to determine their motions (constraints) concisely and visually. In this paper, we propose a geometric algebra (GA) based approach to determine the motions/constraints, mobility and singularity of parallel mechanisms mainly utilizing the geometric and algebraic relations. Firstly, the motions, constraints and their relations are represented by conformal geometric algebra (CGA) formulas in a concise form by employing the characterized geometric elements with $\mathbb{G}_{4,1}$. Secondly, the mobility of parallel mechanism, including its number and property and the axes of motions, not only at origin configuration but also in the prescribed workspace, is obtained by the procedure proposed in this paper. Thirdly, the singularity of parallel mechanism is identified by the two indices proposed in this paper with shuffle and outer products. Finally, a typical example is given to illustrate the motions/constraints, mobility and singularity analysis. This approach is beneficial to kinematic analysis and optimal design of parallel mechanisms, especially for which would be carried out in automatic and visual manner using computer programming languages.

1. Introduction

The first step to analyze parallel mechanisms is to know their mobility, namely the degree of freedom (DoF), including the number and property such as rotation, translation or a combination of both. In the last more than 150 years, sustained efforts have been made to find approach or formula for an efficient and concise calculation of mobility or DoF of any rigid body mechanism [1–3]. There are several dozens of approaches or formulas with various forms presented in the literature, which have been reviewed critically by setting up their origins, similarities and limitations in Ref. [4]. As defined in IFToMM, DoF is the number of independent coordinates needed to define the configuration of a kinematic chain or mechanism [5]. In general, it represents the independent motions except those restricted by the linear-independent constraints of a mechanism. Hence, the number of DoF can be obtained by removing the number of all constraints from the DoF of all moving components of a mechanism, while the constraints mainly depend on the number and type of kinematic joints and their connecting formats and interactive relations [6]. The property of DoF is achieved by discussing all linear-independent constraints imposed on the output of a kinematic chain or a mechanism. In addition, to display the axes of motions is also the important content of mobility analysis except for the number and property. From this point, the essence to determine DoF of any rigid body mechanism is to obtain the relation between motions and constraints of the mechanism [7,8].

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Except for particular parallel mechanisms such as metamorphic mechanisms [9], the mobility number and property of a valid parallel mechanism should remain unchanged over prescribed workspace. However, a parallel mechanism may gain or lose one or more DoFs at its singular configurations [10,11]. In consequence, the singular configurations must be identified and avoided in the kinematic analysis and design of parallel mechanisms. Literature review shows that intensive investigations have been carried out in the past few decades to achieve this goal [12,13]. In general, the singularity of parallel mechanisms can be classified into three types [14]. For the first type of singularity, a parallel mechanism will lose one or more DoFs, which implies that the moving platform becomes immovable in some directions regardless of actuated joint inputs. For the second type of singularity, a parallel mechanism will gain one or more DoFs, which means that the moving platform has uncontrolled movement even all actuated joints are locked. The third type of singularity are defined as the combination of the first and second types of singularity. These singular configurations can be identified by means of analytical, numerical or geometrical approaches [15–17]. Since the overall Jacobian matrix includes the information of both actuations and constraints imposed upon the moving platform, we can directly distinguish the singularity of the overall Jacobian matrix to identify the singular configurations of parallel mechanisms [18]. This approach and its extensions have been applied successfully in the singularity identification of numerous parallel mechanisms [19,20]. It is worth noting that the key point of this approach is to formulate overall Jacobian matrix by exploiting reciprocal property of motions and constraints within each limb.

As mentioned above, the mobility analysis and singularity identification of parallel mechanisms require efficient and concise determination of their motions and constraints. It is known to all that reciprocal property exists between motions and constraints, namely the constrained force (couple) does not work on the rotation (translation) according to the mechanics principle [21]. The motion of parallel mechanism is usually described by a line that represents the motion axis. And the constraint of parallel mechanism is characterized by an action line that satisfies the reciprocal property of motions and constraints. The approaches for determining motions and constraints can be roughly classified into two categories in terms of the observational method and the numerical method [22]. The first category involves seeking the unknown constraints (motions) from the known motions (constraints) merely by observation [23]. For instance, if the motion axes of three known motions are intersected at one point, it is easy to obtain that the corresponding action lines of the three unknown constraints should pass through the same point in order to satisfy the reciprocal property of motions and constraints. It is accurate and efficient but mainly depends on experience. By setting up the matrix composed of all actuations and constraints, the method falling into the second category is to find a null basis vector for the matrix. The core idea is to solve vector equations formulated by the reciprocal property of motions and constraints. The available algorithms include the simplest row echelon, Gram-Schmidt orthogonalization, homogeneous coordinate transformation, the augmented matrix, Gauss Seidel elimination and singular value decomposition [24–27]. Comparing with the observational method, this method converts the issue into a typical mathematical problem, which is able to find the unknown constraints (motions) in a general and systematical way. Whereas, it is pointed out that this method involves high-dimensional matrix calculation and is time consuming for the common situations without screw motions and constraints. Moreover, in most cases, the solutions of these equations are difficult to transfer into motions or constraints with clear physical meaning visually, which is important for illustrating the schematic diagram of parallel mechanism in a visual manner [28].

It is concluded from above-mentioned literature that the descriptions of motions and constraints are expected in visual and concise geometric form whereas the computation of them have to resort to efficient algebraic calculation. The existing approaches deal with this issue merely from geometric or algebraic perspectives, which would inevitably cause problems that does not simultaneously consider the efficient algebraic calculation and visual geometric representation of motions and constraints. Furthermore, to find a systematic and accurate solution for mobility analysis and singularity identification, a geometric algebra (GA) [29] based approach is presented in this paper. Using this approach, only by drawing some auxiliary geometric entities such as points, lines and planes, the unknown constraints (motions) from the known motions (constraints) in common cases is readily and analytically obtained depending on efficient and concise algebraic calculations of GA formulas, and meanwhile the whole procedure is considered to be carried out in a visual manner. Taking advantage of the concise calculation among geometry calculation providing by CGA, common situations of parallel mechanisms are concluded in the paper to help determine unknown motion (constraints) efficiently and accurately. Meanwhile for the cases with helical joints or screw axis, the unknown motions (constraints) could be obtained through solving equations constructed in CGA. Then mobility analysis and singularity identification of parallel mechanisms would be implemented merely using the operation rules of GA such as inner and outer products.

Geometric algebra is also known as Clifford algebra proposed by William Clifford [30]. David Hestenes expressed the geometric interpretation of the algebraic entities and named Clifford algebra as geometric algebra [29]. geometric algebra integrates algebraic systems such as vector algebra, quaternion, Riemann algebra, Lie algebra, complex, screw, etc. algebra system into a unified framework, thus can avoid learning different algebraic languages and their mutual transformations. The subfields of GA could be selected by employing different basic generators according to the application. Since this paper intends to analysis the mobility and singularity of parallel mechanisms in a visual and concise form, the key point of which is to obtain the unknown action lines of constraints (motion axis) by direct geometry entities calculations and linear dependency determination. According to the application, two common geometric algebras are used in this paper, namely, conformal geometric algebra (CGA) [31,32] and $\mathbb{G}_6$. Two superior characteristics of CGA are recognized widely as visual representation and direct calculation for geometric entities. For the former it owns clear geometric and physical meaning while the latter would be carried out coordinate-independently, elegantly and efficiently [33,34]. These make CGA based approach