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Research paper

Design of six-bar function generators using dual-order structural error and analytical mobility criteria

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ABSTRACT

This paper introduces a new approach for the design of planar six-bar mechanisms for the purpose of function generation. The structural error is formulated using the input-output relationship of such a mechanism. In addition to the conventional structural error, its derivative is minimised via numerical optimisation, leading to the novel concept of *dual-order structural error*, which lends itself naturally to a multi-objective formulation of the design problem. Furthermore, analytical conditions for the mobility of the mechanism are derived for two cases: mobility for the full cycle of the crank, and for any given subset of it, along with the identification of the kinematic branches. These conditions help confine the numerical search for the optimal designs to the feasible regions of the design space, leading to a very efficient computational scheme. The results obtained are better in accuracy as compared to the reported results in existing literature. The formulation and results are demonstrated in the context of the Watt-II and the Stephenson-III mechanisms.

1. Introduction

This paper reports new mathematical formulations, which aid the design of planar six-bar mechanisms for the purpose of function generation. In particular, fresh developments are reported with regard to the criteria for the mobility of the Watt-II and the Stephenson-III mechanisms. Also, a new concept, termed hereinafter as the *dual-order structural error*, is introduced. It leads to a multi-objective optimisation formulation for the function generation problem, which, for a number of sample problems studied in the paper, produces results better than those reported in the existing literature, including in the cases where exact synthesis methods were employed.

Traditionally, four-bar mechanisms are used for the purpose of mechanical function generators, following, e.g., the *precision point* approach introduced by Freudenstein [1]. Four-bar mechanisms posses only three independent link ratios (i.e., only three *design variables*), and hence, they can match an arbitrary desired output function *exactly* at three points, at the most. In comparison, the six-bar mechanisms afford much larger design spaces—the three six-bar mechanisms most suitable for function generation, i.e., Stephenson-II, Stephenson-III and Watt-II, have 11 architecture parameters each. Naturally, the six-bar function generators have better potential in terms of accuracy, more so, while approximating highly non-linear functions, which require larger numbers of precision points to describe them accurately over the desired interval of crank motion. This fact was recognised fairly early and there have been sporadic instances of development of such mechanisms as far back as 1940 [2]. However, not many applications and/or theoretical developments have been reported in this regard.

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Recently, an *exact* synthesis of six-bar mechanisms for function generation has been carried out using 8 precision points [3]. The resulting equations, if reduced to a *univariate polynomial*, would have had a total degree of 705432, which would have been practically impossible to solve accurately, even if it could be derived. Hence the set of equations were solved using a *homotopy*-based numerical method, implemented in the special purpose software, `Bertini` [4], leading to 92736 *non-singular* and non-degenerate real solutions. In a more recent work [5], the same group of researches have performed an *exact* synthesis of Stephenson-II mechanisms, having 11 precision points. In this case, starting with a potential solution count of 264241152, a total of 1521037 non-singular solutions are obtained, after running on a 256-core 2.2 GHz computer for 311 h. After some additional post processing, 51 usable solutions were found for the case of hip-motion generator. In the case of Stephenson-III mechanisms, the same team track 55050240 initial solutions to obtain 834441 solutions in 40 h, using a 512-core 2.6 GHz computer on this occasion.

While the recent works reported above represent significant landmarks in the field of computational kinematics, they demand colossal computing resources to obtain the mathematically feasible solutions, not to mention the additional efforts required to identify the set of physically realisable mechanisms from this set. Besides, the use of specific mathematical formulations and consequently highly specialised computational tools are warranted in such analysis. Furthermore, in spite of the potentially huge number of candidate solutions considered at the beginning, a very small number of the final solutions are typically found to be *real* and *feasible*. For example, only 51 feasible solutions were obtained for 11 point synthesis in [5]. Finally, though the formulation and the solutions obtained are *exact* in nature (subject only to certain numerical precision involved), the final check for feasibility/mobility is not analytical, but procedural [6].

An alternative approximate approach, namely, numerical optimisation, can be applied to such situations, which is capable of using simpler formulations and more generic computational tools, while producing results which can be constrained *a priori* to satisfy *any* additional requirements. In the case of the four-bar, several such studies have been reported (see, e.g., [7,8]) for the coupler-curve synthesis problem, even after the nine-point coupler-curve synthesis problem was solved exactly in [9]. For the kinematic synthesis of six-bar mechanisms for function generation, however, the authors were not able to trace a single report pertaining to the optimisation approach (except for a preliminary version of the present work, reported in [10]). This observation may be attributed to the fact that the kinematic formulations of either the objectives or the constraint functions for the six-bar mechanisms are not available in reported literature (to the best of the knowledge of the authors). For instance, a function generator would need to be free of singularities, at least in the desired range of the input. It is hard to incorporate such a requirement in the optimisation process, as no generic “Grashof-like” analytical criteria for feasibility exist in the case of the six-bar mechanisms.

These difficulties have not allowed six-bar mechanisms to be used for function generation, up to their full potential. For instance, a particular class of problems, known as the *double-dwell* synthesis, has been solved using six-bar mechanisms of the Stephenson-III type for many years [11–13]. In this problem, the input is a crank, and the output is a rocker, that has to dwell for finite motions of the crank at both the extremities of its excursion. However, the synthesis of this mechanism has been done traditionally via its reduction to a simpler problem—namely, the coupler-curve synthesis of the four-bar mechanism, having two approximately circular arcs of identical radius of curvature [12].

The present work proposes an optimisation approach to the design of the six-bar function generator mechanisms, in particular, the Watt-II and the Stephenson-III mechanisms. It builds upon several new results, related to the partial and full-cycle mobility, such as explicit conditions on the link lengths, which allow the mechanism to be assembled, and be free of singularities. These new developments allow the identification of combinations of design variables leading to feasible mechanisms with accuracy and certainty. Moreover, they involve only the architecture parameters in such calculations (as opposed to the joint variables), leading to fewer computations.

The formulation of the problem in this work starts with the elimination of the unknown joint variables from the *loop-closure* equations, till only the desired output variable remains. Solutions of this scalar univariate equation (termed as the *forward kinematic univariate* or the *FKU* in brief, following [14]) define the kinematic branches of the mechanism. The branches are identified using the singularity functions, which are derived following the analysis of the constraint Jacobian matrices, as shown in [15]. Each branch is studied independently to identify the potential solutions in it, alleviating the branch-error problem as well in the process. The singularity conditions are converted to polynomials in an algebraic variable representing the crank motion. Characterisation of the roots of these polynomials leads to the identification of the *singularity-free* mechanisms. The problem of computation of assembly constraints is also solved similarly, leading to the identification of link geometries capable of assembling into a feasible mechanism, at a nominal computational expense. Finally, the *structural error*, to be minimised in the optimisation process, is defined in a novel manner. The departure of the generated output function, from the desired output function, is treated as the *zeroth-order* error function, which is in accordance with the standard practice. In addition, a *first-order* error function is defined, which is the derivative of the zeroth order error function. The design method tries to reduce *both* the errors simultaneously, and independently, in what may be called a *dual-order* formulation of the design problem. As the latter objective aids the former, the final results obtained are typically better than in the conventional methods, wherein only the zeroth-order structural errors are considered.

For the solution of the problem formulated as above, a Genetic Algorithm (GA)-based optimiser, namely, `NSGA-II` [16], has been used. Such an optimiser is ideally suited for the problem at hand, since it handles multi-objective problems. It also performs a global exploration, and hence, does not require any initial guess. Furthermore, it is relatively insensitive to the dimension of the design space—which, aided by the confinement of the search for optimal designs to only the feasible regions, allows for satisfactory exploration of the design space in a computationally efficient manner. The results obtained prove to