Mechanism and Machine Theory 117 (2017) 56–77

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Research paper

Design and optimization of a general planar zero free length spring

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ARTICLE INFO

Article history:
Received 4 January 2017
Revised 17 June 2017
Accepted 3 July 2017

Keywords:
Zero free length spring
Null-length spring
Ideal spring
Curved flexure
Shape optimization
Desired force-displacement

ABSTRACT

A zero free length (ZFL) spring is a spring with special properties, which is commonly used in static balancing. Existing methods to create ZFL springs all have their specific drawbacks, which rises to the need of a new method to create such a spring. A method is proposed to design planar ZFL springs with specified stiffness (250–750 N/m) within a certain range (up to 20 mm of displacement). Geometric non-linearities of a curved leaf spring are exploited by changing its shape. The shape is determined by a non-linear least squares algorithm, minimizing the force residuals from a non-linear numerical analysis. Constraints are introduced to help in preventing the spring from intersecting itself during deformation. For three types of springs with different boundary conditions, designs are found with characteristic shapes and maximum force errors less than 1%. A trend is observed between spring size, maximum stress and desired stiffness. New type of ZFL springs can now be designed, which can not only be used in existing applications, but also enables the use of ZFL springs in micro mechanisms.

1. Introduction

The spring is one of the most commonly used physical elements in engineering. A special sub-category in springs is the zero free length spring (ZFL spring), also null-length spring or ideal spring, which is - as the name implies - a spring with zero physical length when no forces are applied. This property results in a spring element which enables a range of special mechanisms to be realized, predominantly in the field of static balancing. To mention a few: Slow wave seismometer [1,2], Anglepoise suspension [3] which are basically balanced arms with a mass [4], zero stiffness mechanisms [5,6], mobile arm support for humans [7,8], camera stabilizer apparatus [9] and in (robotic) manipulators [10,11]. In all these applications any unwanted potential energy differences (resulting from gravity or elastic deformations) are counteracted by one or more ZFL springs, delivering the necessary forces to neutralize the undesired loads.

Before explaining the emerging implementation difficulties of a ZFL spring, it is important to note the properties which define the unique behavior of a ZFL spring. A ZFL spring gets its unique properties from the sole fact that the unstretched (free) length of a linear spring is zero. Observing Fig. 1, a ZFL spring is shown on the left. The length being zero results in the spring pivot and endpoint being coincident. The spring force is now directly proportional to the displacement vector, i.e. the force is in the same direction and its magnitude is proportional to the extension length of the spring ($\boldsymbol{F} = k\boldsymbol{u}$). This in contrast to a normal (non-zero free length linear) spring, where the force is not only dependent on the displacement, but also on the initial position $\boldsymbol{L}_0$ of the spring ($\boldsymbol{F} = k(\boldsymbol{L}_0 + \boldsymbol{u})$). This is shown on the right in Fig. 1. In one dimension there is

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http://dx.doi.org/10.1016/j.mechmachtheory.2017.07.002

List of variables (in order of appearance)

$\boldsymbol{F}$	Reaction/spring force
k	Spring stiffness
$\boldsymbol{u}$	Displacement
$\boldsymbol{L}_0$	Initial length of spring
$\boldsymbol{x}_i$	Control points
N	Number of segments
l_i	Segment length
α_i	Segment relative angle
$\boldsymbol{q}$	Design vector
w	Beam width
h	Beam height
$\boldsymbol{F}_\mathrm{e}$	External force vector
$\boldsymbol{K}$	Stiffness matrix
$\boldsymbol{K}_\mathrm{t}$	Tangent stiffness matrix
$\boldsymbol{F}_\mathrm{i}$	Internal force vector
F_x, F_y	Reaction force component in x and y-direction
F_A	Axial (reaction) force component (aligned with displacement direction)
F_T	Transverse (reaction) force component (force perpendicular to displacement direction)
M	Reaction moment
n_u	Number of displacement steps in one track
n_θ	Number of tracks in different directions
u_min	Minimum displacement (displacement at first sample point)
u_max	Maximum displacement range of the spring
$\tilde{\boldsymbol{F}}$	Desired force
r	Force residual
f_obj	Objective function
α_max	Maximum relative segment angle
l_min	Minimum segment length
l_max	Maximum segment length
$\boldsymbol{r}_0$	Direction vector from the light source node to the shadow segment
$\boldsymbol{r}_\mathrm{s}$	Orientation vector of the shadow segment
$\boldsymbol{R}_{90}$	90° angle rotation matrix
$\boldsymbol{d}_0$	Normal vector of the shadow segment
$\hat{\boldsymbol{n}}_0$	Corrected normal vector of the shadow segment
$\boldsymbol{r}_1$, $\boldsymbol{r}_2$	Direction vectors from the light source node to the corners of the shadow segment
$\boldsymbol{d}_1$, $\boldsymbol{d}_2$	Normal vectors of the corners
$\boldsymbol{n}_1$, $\boldsymbol{n}_2$	Corrected normal vectors of the corners
μ	Direction parameter
$\boldsymbol{R}(\theta)$	Rotation matrix of angle θ
θ_e	Extra outwards rotation of normal vectors
$\hat{\boldsymbol{n}}_1$, $\hat{\boldsymbol{n}}_2$	Normal vectors of the shadow segment corners
f_0, f_1, f_2	Distances to the border of the shadow
g_I	Constraint equation for the shadow-method constraint to prevent self-intersection
N_c	Number of shadow constraint equations
g_II	Constraint equation for contact during displacement
r_g	Penalty function for constraint equations
c_I, c_II	Constraint constants for penalty functions
$f_\mathrm{obj,\,c}$	Constrained objective function
$\boldsymbol{J}$	Jacobian matrix
$\boldsymbol{p}$	Refined control points
ε_A	Maximum relative error of forces in axial direction
ε_T	Maximum relative error of forces in transverse direction
$\varepsilon_\mathrm{shape}$	Error on the shape of the spring

no difference between the two springs, as the reference (zero) point can be chosen freely. But in two or three dimensions the zero point will always be the pivot point on which the spring is fixed.