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Research paper

Design of bistable arches by determining critical points in the force-displacement characteristic

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A R T I C L E I N F O

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A B S T R A C T

The boundary conditions, as-fabricated shape, and height-to-depth ratio of an arch characterize its bistability. We investigate this by modeling planar arches with torsion and translational springs at the pin joints anchored to the ground. A pinned-pinned arch, a special case of the model, has superior bistable characteristics as compared to a fixed-fixed bistable arch, which is also a particular case of the model. Arches with revolute flexures at the ends retain bistable characteristics of the pinned-pinned arches while being amenable for easy fabrication. However, equilibrium equations for such arches become intractable for analytical solution unlike the extreme cases of fixed-fixed and pinned-pinned arches. Therefore, a semi-analytical method for analysis and shape-synthesis of bistable arches with general boundary conditions is developed in this work. This is done by numerically determining critical points in the force-displacement curve. These critical points correspond to switching and switch-back forces and travel between the two states thereby enabling synthesis for desired behavior. We present design and optimization examples of bistable arches with a variety of boundary conditions and as-fabricated shape without prestress. We also propose two approaches to design a new class of asymmetric bistable arches.

1. Introduction

Arches, one of the commonly used structural elements, have been studied extensively in the literature. When subjected to a transverse load, they can undergo a nonlinear snap-through buckling resulting in two force-free equilibrium states. This behavior is a limiting factor while arches are designed for stiffness but desirable in bistable arches that are also known as curved-beam bistable mechanisms. Fig. 1 shows a bistable arch and its force-displacement characteristic. There are three force-free points on the curve: the first and the last points correspond to the stable State 1 and State 2 of the arch and the point in between corresponds to an unstable equilibrium. Bistable arch-based mechanisms are ideal for switches because power is required only for switching from one state to another but not for maintaining the states. Bistable arches also find application in micro-relays, electromagnetic actuators, micro-valves, mechanical memory components, retractable devices, consumer products, circuit breakers, and easy-chairs [1–7]. Further, multistable structures can be produced by combining bistable arches [8].

Bistability in arches can also be achieved by prestress: a buckled column with pinned-pinned boundary conditions is an example. However, precise prestress is hard to realize during bulk-manufacturing and in microfabrication. Therefore, it is

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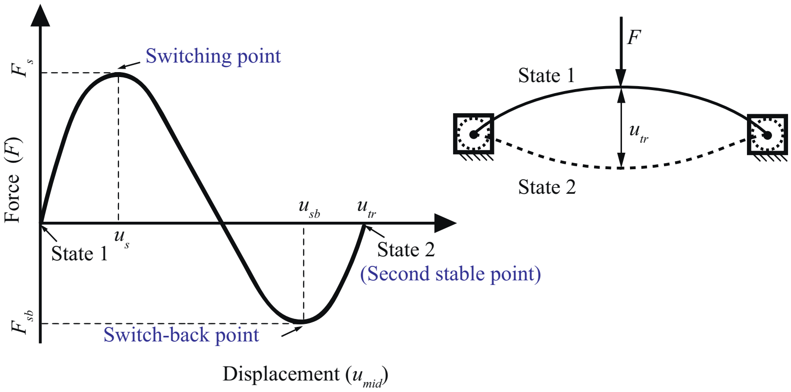

Fig. 1. Typical force-displacement characteristics of a bistable arch.

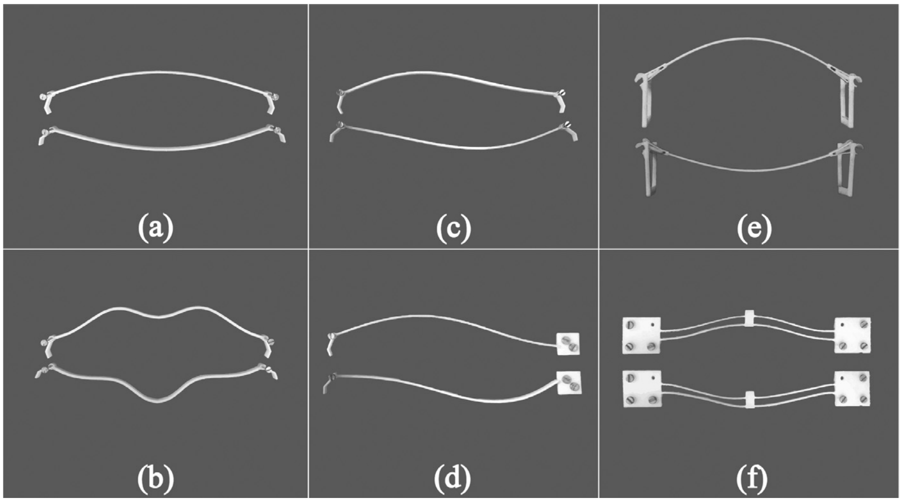

Fig. 2. 3D-printed bistable arches with various boundary conditions and as-fabricated profiles in their two stable states. (a) Pinned-pinned sine arch (b) travel-optimized pinned-pinned arch (c) asymmetric pinned-pinned arch (d) asymmetric pinned-fixed arch (e) split-tube flexure-based arch, and (f) constrained double-cosine arch.

beneficial to pursue fully-compliant monolithic bistable arches that do not rely on prestress for their bistability. One way of achieving bistability without prestress is to make the fundamental buckling mode shape as the as-fabricated arch-shape [9]. Analysis of arches under fixed-fixed boundary conditions has been treated with prestress [10] and without it [9]. Buckling and post-buckling analysis of sine-curved arches, which is also the fundamental mode shape of a straight column with pinned-pinned boundary conditions, has been well studied in two pioneering studies of shallow arches [11,12]; later, this was improved to find the exact snapping loads [13].

A bistable arch with pinned-pinned boundary conditions has advantages over fixed-fixed arches. First, a pinned-pinned arch does not need an interconnected double cosine curve (see Fig. 2(f)) to restrict the asymmetric mode of switching. Second, it has enhanced range of travel between its two stable states and reduced switching force. Third, it has provision for secondary lateral actuation [5]. However, pin joints lead to difficulties in manufacturing at the micro scale and problems in operation due to friction and wear. Therefore, we proposed [14] bistable arches with rotational flexures at the ends with a monolithic design retaining the aforementioned three advantages while easing manufacturing.

As shown in Fig. 1, State 1 is the as-fabricated force-free equilibrium state and State 2 the stressed force-free equilibrium state. Switching force, F_s, is the minimum force required to switch from State 1 to State 2; switch-back force, F_{sb}, is the minimum force required to switch back to State 1 from State 2; and travel, u_{tr}, is the distance the midpoint of the arch moves between the two stable states. Generally, points on the force-displacement curve corresponding to F_s, F_{sb} and u_{tr} are sufficient for designing a bistable arch. Furthermore, bistability of the arch can be assessed by examining the values of F_s and F_{sb}. Hence, we refer to these three points as the *critical points* on the force-displacement curve. We design and optimize bistable arches based on these critical points. This approach simplifies the design methodology to find the optimal bistable design parameters pertaining to size, arch-shape, and cross-section dimensions.