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Research paper

Prevention of resonance oscillations in gear mechanisms using non-circular gears

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ABSTRACT

One of the main disadvantages of gear mechanisms is the occurrence of noise and vibrations. This study investigated the applicability of non-circular gears for preventing resonance oscillations in gear mechanisms. The influence of a small deviation of the gear centrodes from the nominal circles on kinematic and oscillatory characteristics was analysed. It was shown that a larger deviation results in a smaller resonance amplitude due to mesh frequency variability and simultaneously in higher additional dynamic loads on the mechanism. The shape of the gear centrodes was determined which provides a relatively small resonance amplitude with minimum additional dynamic loads. A mechanical device was developed to enable cutting of slightly non-circular gears on a hobbing machine without numerical control.

1. Introduction

Development of reliable, durable and noiseless transmission mechanisms is an important engineering problem. One of the ways of their enhancement is the application of non-circular gears. Mechanisms with non-circular gears are utilised in technical systems in which a mechanical control of speed variation during the working cycle is required. Examples of such systems include mechanical presses [1], textile industry machines [2], high-power starters, hydraulic engines, pumps, flow meters, clocks, and automatic toys. The most commonly encountered shape of non-circular gears is ellipse [3–8], whereas gears with a more complex shape are used to reach special transmission characteristics [9,10]. Non-circular gears have generally involute profile teeth. To increase the transmission load capacity, one also uses Wildhaber-Novikov helical gears with circular arc teeth [11–13].

The studies investigating non-circular gears are usually aimed at resolving the following problems:

- determination of the gear centrodes that provide the required kinematic properties, for instance: smooth variation of the gear ratio [14,15], cyclic speed variation [16], motion synchronisation [4], reduction of the working cycle duration [1], arbitrary variation of the gear ratio [17,18], prescribed relationship between the input and output angular displacements [19], intermittent motion [20];

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- determination of the gear centrodes that allow to reduce dynamic loads on the mechanism [21–23];
- synthesis of the gear tooth profile with optimum strength properties [8];
- gear design automation and enhancement of the gear cutting technology [5–7,11,12,24–26].

The above literature review shows that non-circular gears were investigated with respect to the kinematic and dynamic behaviour, strength properties, and cutting technology. However, there are still unexplored aspects of the application of non-circular gears. One of them is the influence of the gear non-circularity on the occurrence and intensity of noise and vibrations in gear mechanisms.

The main reason for gear noise is considered to be the transmission error [27] defined by Welbourn [28] as 'the difference between the actual position of the output gear and the position it would occupy if the gear drive were perfectly conjugate'. The causes of the transmission error are deformations of the gears (contact areas in the gear mesh, gear teeth, gear blanks), deformations in the mechanism (shafts, bearings, casing), geometrical errors (gear teeth profiles, gear centrodes, position of the gear carrying shafts, position of the bearings in the casing), gear teeth wear, etc. Because of the transmission error, the gear teeth come into mesh with a dynamic impact [29], generating an excitation force that acts on the mechanism with the mesh frequency and its harmonics [30]. If the mesh frequency or any of its harmonics coincides with one of the natural frequencies of the mechanism, resonance effects will occur, including additional dynamic loads on the mechanism [31] and noise emissions [32].

The aim of the present study was to theoretically investigate the applicability of non-circular gears for preventing resonance oscillations in gear mechanisms.

Notation

f_0	excitation amplitude	$\varepsilon_{1,2}$	gear angular acceleration
i	gear ratio, $i=\omega_2/\omega_1$	$\varepsilon_{\max}$	maximum absolute value of ε_2, $\varepsilon_{\max}=\max\lvert\varepsilon_2\rvert$
j	number of the gear ratio variation cycles per turn of the input gear	ξ	coefficient of the gear ratio variation smoothness, $\xi>1$
k	parameter, $k=j/z_1$	π	Pi number, $\pi\approx 3.14$
r	nominal radius of the input gear centrode	τ	dimensionless time variable, $\tau=\omega_\mathrm{n}t$
$r_{1,2}$	instant radius of the gear centrode	$\varphi_{1,2}$	gear angular displacement
t	time variable	ω	dimensionless velocity of the input gear, $\omega=z_1\omega_1/\omega_\mathrm{n}$
u	gear teeth ratio, $u=z_2/z_1$	$\omega_{1,2}$	gear angular velocity
x	displacement	ω_n	angular natural frequency
$z_{1,2}$	gear teeth number	ω_e	dimensionless excitation frequency
B	maximum relative deviation of r_1, $B\ll 1$	ω_m	angular mesh frequency, $\omega_\mathrm{m}=r_1z_1\omega_1/r$
X	dimensionless displacement, $X=x\omega_\mathrm{n}^2/f_0$	$\blacksquare_1$	quantity related to the input gear
δ	speed fluctuation coefficient	$\blacksquare_2$	quantity related to the output gear

2. Definition of the gear centrodes and the gear ratio

We consider a meshed pair of non-circular gears. The instant radii r_1 and r_2 of the input and output gear centrodes, respectively, are defined by the following functions [13]:

$$r_1=r\left(1+\frac{B\sqrt{\xi^2-1}\ \sin(j\varphi_1)}{\xi+\cos(j\varphi_1)}\right);\ r_2=r\left(u-\frac{B\sqrt{\xi^2-1}\ \sin(j\varphi_1)}{\xi+\cos(j\varphi_1)}\right) \tag{1}$$

The gear ratio i has the form

$$i=\frac{\omega_2}{\omega_1}=\frac{r_1}{r_2}=\frac{\xi+\cos(j\varphi_1)+B\sqrt{\xi^2-1}\ \sin(j\varphi_1)}{u(\xi+\cos(j\varphi_1))-B\sqrt{\xi^2-1}\ \sin(j\varphi_1)}\approx\frac{1}{u}+\frac{B(1+u)\sqrt{\xi^2-1}\ \sin(j\varphi_1)}{u^2(\xi+\cos(j\varphi_1))} \tag{2}$$

Here φ_1 is the input gear angular displacement; r is the nominal radius of the input gear centrode; u is the gear teeth ratio, $u=z_2/z_1$; z_1 and z_2 are the teeth numbers of the input and output gears, respectively; ω_1 and ω_2 are the angular velocities of the input and output gears, respectively; B is the maximum relative deviation of r_1, $B\ll 1$; ξ is the coefficient of the gear ratio variation smoothness, $\xi>1$; j is the number of the gear ratio variation cycles per turn of the input gear, $j\in\{1,\ 2,\ 3,\ \dots\}$. The output gear angular displacement φ_2 is determined by integrating Eq. (2):

$$\varphi_2=\int_0^{\varphi_1} i\ d\varphi_1\approx\frac{\varphi_1}{u}-\frac{B(1+u)\sqrt{\xi^2-1}}{ju^2}\ \ln\left(\frac{\xi+\cos(j\varphi_1)}{\xi+1}\right)$$

Fig. 1 plots the gear centrodes and the gear ratio i for various values of j. In order to emphasise the curvature of the gear centrodes, B is set to a relatively large value.

The coefficient ξ affects the shape of the gear ratio function, as illustrated in Fig. 2. At ξ close to 1, i changes abruptly (see the curve $\xi=1.1$). With increasing ξ, the variation of i becomes smoother. At large values of ξ, i is almost a sinusoid (see the curve $\xi=10$).