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Research paper

A proposal for a formula of absolute pole velocities between relative poles

J. Viaña*, V. Petuya

Department of Mechanical Engineering, University of the Basque Country UPV/EHU, Alameda de Urquijo s/n, 48013 – Bilbao, Spain

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ABSTRACT

In this paper is proposed a vectorial equation that relates the absolute pole velocities of three moving rigid bodies with a planar motion of general type. From this equation, it is possible to obtain a relation between the pole velocities of the three mathematical points, related between them by the Aronhold-Kennedy Theorem. The formula allows the calculation of one of the pole velocities from the other two, being known the angular velocities and accelerations of the moving bodies. It is applicable regardless of whether the instantaneous centers (poles) are located on physical points on the linkage or not. Illustrative examples of the application of the formula on representative planar linkages are included. In the final section, is discussed a similar concept associating a mathematical point to the curvature centers of a point's path, so called centroma.

1. Introduction and approach to the problem

The instantaneous center (pole) of a rigid body with planar motion has the property of being a physical point that instantaneously (or even permanently) has no speed. Consequently, the planar motion of a rigid body can be studied as a sequence of differential rotations about an axis perpendicular to the plane containing the pole.

This point has a physical-mathematical duality. The physical point, which belongs to the moving body, is the one whose velocity is zero. But the mathematical point has the so called pole velocity, in the direction of the centrode tangent. In this paper, the velocities corresponding to linkage joints or physical points will be identified by the vector $\boldsymbol{v}$ while those corresponding to pole velocities will have the vector $\boldsymbol{u}$ associated.

The poles can be absolute if they have null velocity with respect to the frame, or relative having null velocity in a relative motion between two bodies. Authors like Hunt [1] have already investigated in the relations obtained when referring to the study of the pole in the relative motion. The present research is based on [2], where the planar motion bases and the motion of a mechanism are explained with the geometry that underlies behind.

All the examples analysed and the calculations done, have been simulated and verified using the GIM research and educational numerical software [3]. This software is being developed by the COMPMECH research group of the University of the Basque Country - UPV/EHU (www.ehu.es/compmech). Also in [4], a relationship for computing several angular velocities of rigid bodies can be found.

The curvature theory and the envelope theory are presented in detail in [5,6]. In references [6,7] the general form of Euler-Savary equation, together with the Aronhold theorem and Hartmanś construction are explained. The inflection circle

* Corresponding author.
E-mail addresses: javiervianaperez@gmail.com (J. Viaña), victor.petuya@ehu.es (V. Petuya).

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Nomenclature

A_i	Geometric point A of moving body i.
P_{ij}	Instantaneous center (pole) of velocity of moving body i with respect to moving body j.
$\mathbf{u}_{ij}$	Absolute pole velocity of P_{ij}.
$\boldsymbol{\omega}_i$	Angular velocity of moving body i.
$\boldsymbol{\omega}_{i_{r_j}}$	Angular velocity of moving body i relative to moving body j.
$\rho_{f\mathbf{i}}$	Radius of curvature of the fixed centrode of moving body i.
$\rho_{m\mathbf{i}}$	Radius of curvature of the moving centrode of moving body i.
$\boldsymbol{\alpha}_i$	Angular acceleration of moving body i.
$\boldsymbol{\alpha}_{i_{r_j}}$	Angular acceleration of moving body i relative to moving body j.
$\boldsymbol{v}_C$	Velocity of physical point C.
O_A	Centroma of A (mathematical point associated to the centre of curvature of the path of point A).
$\mathbf{r}_{IJ}$	Position vector relating points I and J.

and the cuspidal circle are explained in [8] while in [9] is clearly presented the concept of instantaneous center and the relative velocity field. In [10,11] the kinematic analysis of complex mechanisms is presented.

In references [12,13] the concepts of relative angular velocity and acceleration are introduced and applied in representative examples. In [14], new formulas for the first and second time derivatives of 2×2 transforms based on the Cayley-Klein parameters are derived. Based on these, an extension to the computation of velocities and accelerations of the kinematic analysis proposed by Denavit [15] is presented.

Reference [16] investigates the instantaneous spatial higher pair to lower pair substitute-connection which is kinematically equivalent up to acceleration analysis for two smooth surfaces in point contact. In [17] using the contact kinematics equations of the enveloping curves, is shown how the theorem on coordinated centers is valid for a position in which the instantaneous relative angular velocity is zero [18]. This is possible since the approach does not make any reference to the polodes.

Fig. 1 shows the case of pure rolling motion between two disks being 1 the frame. The null velocities of the absolute poles P_{12}, P_{13}, the velocity of the relative pole P_{23} and the absolute pole velocities $\mathbf{u}_{12}$, $\mathbf{u}_{12}$ y $\mathbf{u}_{23}$ are depicted.

Being $\boldsymbol{\omega}_i$ the angular velocity of the moving body *i*, ρ_f and ρ_m the radius of curvature of the fixed and moving centrodes respectively and using the Euler-Savary formula, u_{12} and u_{13} are,

$$u_{12} = \frac{\omega_2}{\frac{1}{\rho_{f2}} - \frac{1}{\rho_{m2}}}; \quad u_{13} = \frac{\omega_3}{\frac{1}{\rho_{f3}} - \frac{1}{\rho_{m3}}} \tag{1}$$

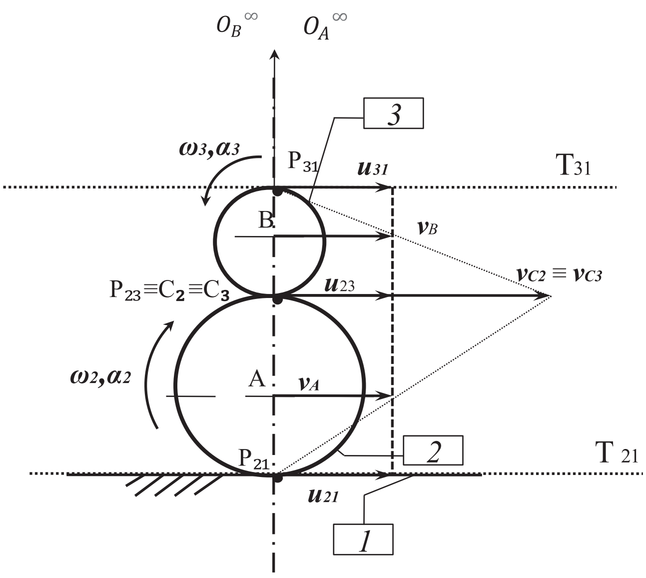

Fig. 1. Rolling motion between two disks.