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Numerical analysis of theoretical flow in external gear machines

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ABSTRACT

With their advantages of low-cost, high-reliability and simplicity, external gear machines (EGMs) are among the most common pumps or motors in high pressure applications. As all positive displacement machines, EGMs are characterized by a substantial flow non-uniformity, which is given by the gear meshing and results in vibrations and noises. Several analytical methods are available but require the knowledge of the tooth profile, involute or cycloidal, and are not directly applicable to unconventional gears. This paper presents a numerical approach for the analysis of the theoretical flow through spur gear EGMs which does not require a-priori knowledge of the gear profile. Starting from a CAD input, the proposed approach automatically determines control volumes for every angular position of the gears to describe the displacing action realized by the unit. After describing the method, the paper shows its application to different geometric morphologies for the gear profiles to calculate the theoretical flow rate. The verification of the numerical model is demonstrated by comparing its results with those provided by the analytical methods available in literature for the case of involute gears.

1. Introduction

External gear machines (EGM) are widely used as pump or motor in many fluid power and motion control applications, in engine lubrication, in fuel injection, in washing and fluid transport systems. Key factors for their commercial success are the low manufacturing cost, good energy efficiency, high-reliability and simplicity of operation. Although classified as positive displacement machines, the displacing action of EGMs is not solely realized by a physical displacement of the boundary of a certain fluid mass, as it happens for piston machines. In fact, the tooth space volumes that function as displacement chambers are entirely surrounded by solid boundaries only in a limited region of the meshing process. This region corresponds to the portion of the meshing process in which the tooth space volumes are trapped between points of contact. For typical designs of the gear profile, this is represent only a small portion of the volume variation realized by each displacement chamber.

The complexity of analyzing the volume displaced by an EGM with the gear rotation captured the attention of many researchers for decades. The work by Bonacini [1] is most likely the first published study that presented the application of the so called “energy method” for calculating the instantaneous flow through an EGM. This method equates the hydraulic work with the input shaft energy to derive an expression for the displaced volume, for a given law of variation of the contact point between gears. In his work, Bonacini also presented a “geometric approach” based on the analysis of variations of a control volume defined at one of the ports with the gear rotation, and demonstrated the equivalency of the two approaches.

The energy method was successfully taken as reference method of study of the displaced volume in EGMs by other authors, such as in the book by Ivantysyn and Ivantysynova [2] or the work by Devendran [3]. Similarly the geometric method was used in other studies, such as by Huang and Lian [4] who derived an expression very similar to [1,5] who derived a formula with a higher order

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Abbreviations: EGM, External Gear Machine; DC, Displacement Chamber

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Nomenclature

b	whole thickness of gear (mm)
h_a	addendum value [mm]
h_d	deddendum value [mm]
i	center distance (mm)
m	module of gear (mm)
Q	flow rate (mm^3/s)
r_a	outer radius of gear (mm)
r_b	radius of base circle (mm)
r_p	radius of pitch circle (mm)
t	time (s)
u	distance from contact point to pitch point (mm)
V_{var}	variable volume (mm^3)
V_1	volume of DC1 (mm^3)
V_2	volume of DC2 (mm^3)
x	correction (offset) factor (1)
z	number of teeth (1)
z_1	number of DCs in Gear 1 that are increasing volume [1].
z_2	number of DCs in Gear 2 that are increasing volume [1].

Greek symbols

α	designed pressure angle of gear (deg)
α'	working pressure angle of gear (deg)
ω	shaft speed (rad/s)
γ	base pitch (mm)
θ	angular position of gear (deg)
ϕ	angle between inward tangential vector and the vector pointing to the center (deg)
ϵ	minimum segment length below which backlash is considered negligible (mm)

Subscript

i, j, k, p, q	DC index
in	volume connected to inlet
out	volume connected to outlet
sf	single-flank
df	dual-flank

accuracy.

The main outcome of the mentioned past studies was an analytical expression for the instantaneous volume displaced by an EGM, which correspond to the flow rate, if leakages and compressibility effects can be neglected (*theoretical flow rate*). The knowledge of the instantaneous flow rate permits to derive the flow irregularity, which is an important parameter for the EGM, often used to quantify the fluid-borne noise as well as the potentials of the unit for inducting mechanical vibrations within the entire hydraulic system. Several commercially successful "silent" EGMs have gears whose profile is designed to specifically reduce the theoretical flow rate: examples are documented in [6,7]. This proves the importance of creating methods suitable to describe the theoretical flow rate of an EGM; such tools can assist the designer in the formulation of silent and vibration-less EGMs.

However, some limiting factors of abovementioned approach limit their application for design purposes. In fact, in some cases a single pair of tooth at a time is assumed to contribute to the overall process of flow displacement (this is not a good assumption for gears that realize a contact ratio significantly different from one). But, most importantly, all the approaches are based upon the knowledge of the analytical formulation for the tooth profile, either of an involute- or of a cycloidal-type. Effect of tooth profile corrections required to guarantee a proper operation of the EGM, or study of novel gear profiles, such as the cosine profile introduced by Luo et al., [8], would require an entirely new analytical derivation of the theoretical flow ripple.

To address this problem, and propose a general method for the analysis of the kinematic volume variations realized by an EGM in operation, the authors have formulated a numerical method that does not require a predetermined morphology for the gear profiles. This method, which is the main subject of this paper, permits a quick evaluations of the theoretical flow rate of an EGM as a function of the gear geometry.

The proposed model can complement the existing CFD models available for the analysis of EGM, such as the models proposed by Castilla et al. [9] or commercial software such as Pumplinx [10]. Firstly, it can serve as s reference for the theoretical displaced flow related to the pump geometry. Additionally, its simulation swiftness makes this model more suitable than detailed 3D-CFD models for initial parametric studies. The formulation used to describe the displacement chambers (DCs) in an EGM can also serve for the

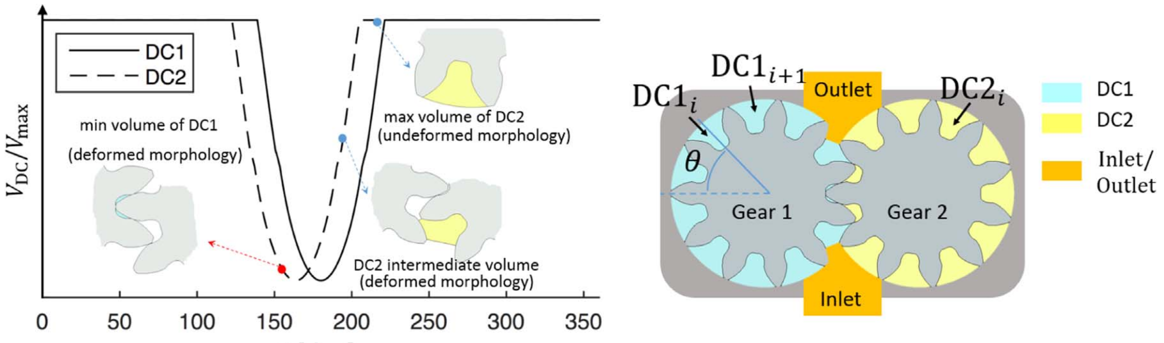

Fig. 1. The DC volume curve and the different shapes that each DC assumes at minimum volume, maximum volume, and intermediate volume conditions.