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A way of relating instantaneous and finite screws based on the screw triangle product

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ABSTRACT

It has been a desire to unify the models for structural and parametric analyses and design in the field of robotic mechanisms. This requires a mathematical tool that enables analytical description, formulation and operation possible for both finite and instantaneous motions. This paper presents a method to investigate the algebraic structures of finite screws represented in a quasi-vector form and instantaneous screws represented in a vector form. By revisiting algebraic operations of screw compositions, this paper examines associativity and derivative properties of the screw triangle product of finite screws and produces a vigorous proof that a derivative of a screw triangle product can be expressed as a linear combination of instantaneous screws. It is proved that the entire set of finite screws forms an algebraic structure as Lie group under the screw triangle product and its time derivative at the initial pose forms the corresponding Lie algebra under the screw cross product, allowing the algebraic structures of finite screws in quasi-vector form and instantaneous screws in vector form to be revealed.

1. Introduction

The instantaneous screw [1] has been considered as a powerful tool and extensively applied in analysis and design of serial and parallel mechanisms. By taking twist of an end link as a linear combination of instantaneous screws produced by joints, Hunt [2] and Duffy [3] developed a method to formulate Jacobian matrix. Angeles [4] presented an alternative way to do so by taking all actuation wrenches as instantaneous screws. These works were extended by Joshi and Tsai [5], Huang and Liu [6] in dealing with lower mobility serial and parallel mechanisms, resulting in the overall Jacobian and generalized Jacobian matrix that allow velocity, force, stiffness and rigid body dynamic analyses to be unified using instantaneous screw theory. Another use of screw theory in analysis and synthesis of robotic mechanisms is mobility analysis and type synthesis. Regarding small and finite displacements or velocities and constraint forces of a mechanism in a systematic frame of a screw system and its reciprocal system, Dai and Rees Jones [7] revealed the interrelationship and intersection of all screw systems and their reciprocal systems by introducing set theory in screw theory. In the meantime, they [8] developed a linear algebraic procedure to obtain a reciprocal screw system and its basis in a null space construction using cofactors from a screw algebra context. These theories were directly used in mobility analysis of different types of parallel mechanisms, including those having overconstraints [9]. Utilizing annihilator property between a screw system and its reciprocal system, Huang and Li [10], Fang [11], Kong [12] and colleagues proposed a number of simple and effective approaches for

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type synthesis of various parallel mechanisms. However, since instantaneous screws are not valid to represent finite motions, final verification is required to ensure consistency between finite and instantaneous motions [13–15].

In order to describe finite motions precisely, the finite screw was proposed. Investigation into finite screws can be traced back to the pioneering work of Dimentberg [16,17] and Roth [18] in dealing with composition of finite motions of a rigid body, leading to development of the screw triangle product that provides a sound geometric interpretation to the Rodrigues formula with dual angles [17]. Investigations into finite screws were developed by Roth [18], Parkin [19], Huang [20,21] and Dai [22,23]. They proposed a simple and widely accepted form of finite screws that can be analogously expressed as a quasi-vector with six algebraic entries. A finite screw in this form looks extraordinarily similar to an instantaneous one even if it is nonlinear in nature. This form allows the linear format of a screw triangle to be made in such a way that the composition of two finite screws can be expressed as the sum of five meaningful terms [21,24]. Based upon the algebraic characteristic of finite screws, Dai, Holland and Kerr [22] and Dai [25,26] revealed the relationship between finite screws and instantaneous screws through solving the eigenscrew and derivative of the finite displacement screw matrix [25]. Utilizing finite screw theory, finite motion analyses of different geometrical elements, such as points, lines and planes, as well as simple open loop and closed loop mechanisms are carried out by Huang [20,27], Hunt [28].

It has been a desire to develop a theoretical package that enables analysis and synthesis of robotic mechanisms to be integrated into a unified and consistent process [29]. This issue needs to relate nonlinear finite to linearized instantaneous motions of rigid body systems. Hence, a preliminary and essential step to achieve the aforementioned goal is to develop a general and effective method that enables the description, formulation and operation of finite and instantaneous motions to be implemented under a consistent and unified mathematical tool, a fundamental and challenging issue in the field of mechanisms and robotics. The methods available at hand can be roughly divided into three categories, i.e. matrix group based method, dual quaternion based method and screw theory based method.

The matrix group based method can be traced back to the Erlangen program proposed by Klein [30]. By utilizing matrix groups to describe finite motions of rigid body systems, Brockett [31] applied the exponential map between Lie group SE(3) and Lie algebra se(3) to relating models for finite motions to that for instantaneous motions [32]. However, two barriers are encountered in the use of matrix groups for finite motion composition. The first barrier arises from implementation of matrix groups for affine transformations where finite motions of a rigid body cannot be directly represented by Chasles' axis [17] as well as by the angular and/or linear displacement about the axis, leading to a complicated description of rigid body motion. The second barrier comes from that the finite motion composition cannot be algebraically derived by Baker-Campbell-Hausdorff formula [33]. Consequently, motion patterns of a number of parallel mechanisms cannot precisely be described using the existing matrix group based method since they can no longer be represented by products of several Lie subgroups [34].

The dual quaternion based method can be traced back to description of rotations of a rigid body by means of Euler's four-square identity, Euler-Rodrigues parameters [23,35] and Hamilton quaternions [36]. Perez and McCarthy [37] seem to be the first to use the dual quaternions to do analyses for finite and instantaneous motions of serial kinematic chains. In their work, unit dual quaternions and unit pure dual quaternions were used for describing finite and instantaneous motions, for the algebraic structure of the former is a double cover of Lie group SE(3) whose Lie algebra in turn constitutes the latter as by Selig [38]. With the aid of group theory, Selig [39,40] and Dai [26,35] investigated the algebraic properties of the exponential and Cayley maps between unit dual quaternions and unit pure dual quaternions. By introducing the notation of high-dimensional Clifford algebra, Selig [40] and Featherstone [41] extended the dual quaternions representation to deal with rigid body dynamics. However, a unit dual quaternion is not the simplest form of a rigid body motion. The redundancy in dual quaternion representation may cause complexity in analytical expressions of the finite motion operations. In addition, the Rodrigues formula with dual angles is not the simplest form of the Baker-Campbell-Hausdorff formula when it is applied to composition of finite motions of a rigid body [42].

The screw theory based method depends on the development of instantaneous screws and finite screws. Considering the algebraic characteristic of finite screws, Dai [22] demonstrated the relationship between finite displacement screw operation and the different matrix representations of SE(3) elements as well as quaternions [25,35]. By solving the eigenscrew and derivative of finite displacement screw matrix, Dai [25] formulated the eigen and differential mappings between finite screws and instantaneous screws. Meanwhile, the consistency between these mappings and the exponential mapping of matrix Lie group/Lie algebra or the Euler-Rodrigues formula was revealed [25,26,35]. The correspondence between vector subspaces and screw systems was discussed by Huang, Sugimoto and Parkin [43] to differentiate finite screw systems from screw systems arising from instantaneous kinematics and statics. Although the composition of finite motions can be expressed as a screw triangle product of finite screws in quasi-vector form [21,24], and that of instantaneous motions as the linear combination of instantaneous screws in vector form, the algebraic structures of two kinds of screws remain an open issue to be investigated.

Addressing on the need to integrate models for analysis and synthesis of robotic mechanisms in a unified mathematic framework, this paper intends to reveal algebraic structures of finite screws in a quasi-vector form and instantaneous screws in a vector form. The paper is organized as follows. Having a brief review of the state-of-the-art of finite and instantaneous screw theory in Section 1, Section 2 presents the derivation of finite screws from dual quaternions and addresses the description of finite screws in quasi-vector form and instantaneous screws in vector form. Section 3 explores the associativity and derivative properties of the screw triangle product, resulting in that the set of finite screws is an associative and differentiable algebraic structure. This leads to a vigorous proof in Section 4 that the entire set of finite screws forms a Lie group under the screw triangle product and its time derivative at the identity forms the corresponding Lie algebra under the screw cross product before the conclusions are drawn in Section 5.