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Position and orientation characteristics of robot mechanisms based on geometric algebra

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ABSTRACT

The operable representation of robot mechanisms is the key to the computerization of type synthesis of parallel mechanisms (PMs). Geometric algebra, an effective mathematical tool for geometric representation and computation, is introduced to describe the position and orientation characteristics (POC) of robot mechanisms, including serial and parallel mechanisms, in this paper. According to the correspondence between the POC of the joint axis and the motion output characteristics of the moving link, the POC union is defined by the outer product operation for the serial kinematic chain and the POC intersection is defined by the shuffle product operation for the parallel kinematic chain. In this paper, a moving coordinate system independent of the position of a mechanism is established for the expression of the joint axes. The direct symbolic algorithm for the motion output characteristics of robot mechanisms, including union and intersection, is proposed and validated via several case studies of 3R open chain, Sarrus linkage, 3-RCC PM and 3-RPS PM. The analysis procedure shows intuition and explicitness that is suitable for computer-aided derivation of the POC set of a mechanism.

1. Introduction

The digital type synthesis of robot mechanisms, especially parallel mechanisms (PMs), has been a critical part of developing software for innovative design of robot mechanisms. Gao and Meng [1] describe a framework for computer aided type synthesis of PMs and have developed the integrated software based on the G_F set. Ding [2] presents the character strings to represent the complete topological information of PMs for computer processing of type synthesis of both symmetrical and asymmetrical 5-DOF PMs. Yang [3–5] proposes three concepts, including position and orientation characteristics (abbreviated as POC hereafter) set, single open chain (SOC) unit and dimensional constraint types, to describe mechanism topology structure using symbols and applies them to type synthesis of various PMs. There is still a long way to go to achieve automatic mechanisms synthesis using this method. Operable representation of robot mechanisms is the key to this problem.

The effective description of robot mechanisms depends on the characterization of joint axes or the representation of connection relationship between links. In addition to the Screw Theory and Lie Algebra, some other geometric and algebraic tools have been used for the analysis of mechanisms. Grassmann geometry [6] and Grassmann-Cayley algebra [7,8] have been applied to singularity analysis of some types of PMs. Husty and et al. [9] introduce the Study's kinematic mapping of the Euclidean group to reveal global kinematic behavior properties of 3-RPS parallel manipulator. Based on this approach, Nurahmi [10] describes a 4-RUU parallel manipulator by a set of constraint equations and computes the primary decomposition which can deal with the characterization of

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the operation modes. Kong [11,12] makes Euler parameter quaternions be classified into various cases according to the number of their constant zero components and deals with reconfiguration analysis of PMs.

Geometric algebra (GA) [13–15], a mathematical framework for the geometric representation and computation, provides the geometric meaning of the algebraic expressions. Tanev [16,17] performs the screw in the geometric algebra $\mathbf{G}^6$ and obtains the singularity conditions of PMs. Li [18] adopts the twists and wrenches in the form of geometric algebra to describe the branches of PMs and proposes a mobility analysis approach for limited-DOF PMs based on the outer product operation. Bayro-Corrochano [19], Hildenbrand [20], Fu [21], Wei [22] have applied GA to the solutions of inverse kinematics problem of the serial robot. Kim [23] performs a geometric singularity analysis and avoidance using conformal geometric algebra. These researches indicate that geometric algebra is a powerful tool and has the advantages of conciseness and intuition for the application in robotics.

This paper aims to realize the mathematical description and automatic derivation of the position and orientation characteristics (POC) of robot mechanisms. Based on the six dimensional expression of the spatial line, the position and orientation characteristics of the joint axis and the motion output characteristics of the moving link are described in a unified form. A moving coordinate system independent of the position of a mechanism is established, the outer and shuffle products are extended to the operations for serial and parallel kinematic chains. Furthermore, by using the GA based POC description, a symbolic algorithm is proposed to obtain the motion output characteristics of the robot mechanism. This method greatly simplifies the analysis procedure.

The rest of this paper includes four sections, which are organized as follows. Section 2 presents the foundation of geometric algebra and its utilization in this paper. Section 3 introduces the POC description of kinematic joints and the operation for serial and parallel kinematic chains. Section 4 takes several mechanisms for instances to illustrate the proposed method. Finally, the conclusions and future work about this paper are discussed in Section 5.

2. Geometric algebra and vector space

To describe the position and orientation of a joint axis, geometric algebra (GA) is applied to the expression of the spatial line. The fundamental operator in GA is geometric product [15], including the inner product and the outer product, which can be expressed as

$$ab = a\cdot b + a \wedge b \tag{1}$$

For 3-dimensional Euclidean space, geometric algebra $\mathbf{G}^3$ has a group of orthogonal basis vector $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$. Its corresponding conformal model $\mathbf{G}^{4,\ 1}$, which is called as conformal geometric algebra [24], has five basis vectors $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3, \boldsymbol{e}_0, \boldsymbol{e}_\infty\}$ that $\boldsymbol{e}_0$ represents the origin and $\boldsymbol{e}_\infty$ represents the infinity, which have the following relationship

$$e_1^2 = e_2^2 = e_3^2 = 1,\ e_0^2 = e_\infty^2 = 0 \text{ and } e_0\cdot e_\infty = -1 \tag{2}$$

This 5D conformal geometric algebra for 3D space makes the points, spheres and other geometric entities easily be represented as vectors in a unified form and handles conformal transformations easily [15]. In $\mathbf{G}^{4,\ 1}$, the spatial line $\boldsymbol{L}$, denoted by $\boldsymbol{L}^{4,\ 1}$, can be generated by the intersection of two planes $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$ via the outer product, as shown in Fig. 1, which is expressed as

$$\begin{aligned}\boldsymbol{L}^{4,1} = \boldsymbol{\pi}_1 \wedge \boldsymbol{\pi}_2 &= (a_1\boldsymbol{e}_1 + b_1\boldsymbol{e}_2 + c_1\boldsymbol{e}_3 + d_1\boldsymbol{e}_\infty) \wedge (a_2\boldsymbol{e}_1 + b_2\boldsymbol{e}_2 + c_2\boldsymbol{e}_3 + d_2\boldsymbol{e}_\infty) = (b_1c_2 - c_2b_1)\boldsymbol{e}_{23} + (c_1a_2 - a_1c_2)\boldsymbol{e}_{31} + (a_1b_2 - a_2b_1)\boldsymbol{e}_{12}\\ &+ (a_1d_2 - a_2d_1)\boldsymbol{e}_{1\infty} + (b_1d_2 - b_2d_1)\boldsymbol{e}_{2\infty} + (c_1d_2 - c_2d_1)\boldsymbol{e}_{3\infty}\end{aligned} \tag{3}$$

Where (a_i, b_i, c_i) refers to the 3D normal vector of the plane $\boldsymbol{\pi}_i$ and d_i refers to the distance from the plane $\boldsymbol{\pi}_i$ to the origin. The $\boldsymbol{e}_{ij}$ is a 2-blade, which denotes the 2D directional plane, as shown in Fig. 2.

Based on Eq. (3), a line in space can be identified by using these six 2-blades, which can be treated as 1-vector in the 6-dimensional vector space. This space corresponds to the geometric algebra $\mathbf{G}^6$ [16], which consists of six basis vectors $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3, \boldsymbol{e}_4, \boldsymbol{e}_5, \boldsymbol{e}_6\}$ that obey the following rules

$$\mathrm{e}_i \wedge \mathrm{e}_i = 0 \tag{4}$$

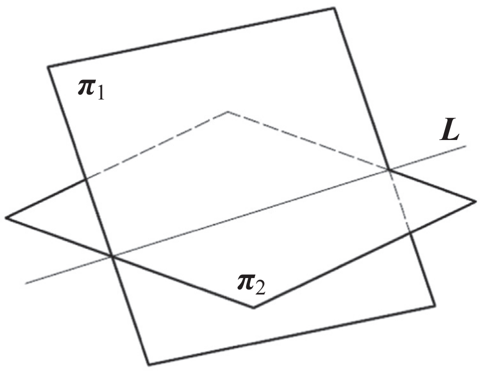

Fig. 1. Intersecting line of planes.