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Removal of singularities in the inverse dynamics of parallel robots

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ABSTRACT

It is commonly claimed in the literature that if the dynamic equations are made consistent at a type II singular configuration, then a parallel robot can pass through this configuration while the actuator forces remain bounded. However, in contrast to the existing literature, the present paper proves that the consistency of the dynamic model is not sufficient alone within this context, and derives the additional necessary conditions for this purpose. It is then shown that under these new conditions, type II singularities of parallel manipulators can be removed to guarantee finite and continuous inverse dynamics solutions while fully utilizing the workspace. These are achieved through a novel analysis of type II singularities of parallel robots based on the limit concept. Another contribution of the paper is the proposal for definitions of natural and actual orders of type II singularities.

1. Introduction

Despite notable advantages over their serial counterparts [1], parallel robots are also known for being practically confined to a rather small portion of their workspace due to type II singularities (as classified by [2]), around which the inverse dynamics solution grows without bounds and the controllability is lost [3,4]. Their type I singularities, on the other hand, are generally encountered at the boundaries of the workspace [2], and for this reason do not result in a serious practical limitation.

An immediate solution is the use of redundancy [5], a recent example of which can be seen in [6]. Another recent proposal is to sacrifice the control of one of the degrees of freedom of the robot [7].

A more desirable and cheaper solution might be to enable non-redundant parallel robots to directly pass through type II singularities in a controllable and stable manner, i.e. while the necessary actuator forces remain bounded. Actually, this is not impossible under certain conditions, as partly revealed by the following works: By assuming that the transformation matrix mapping the actuator forces to the task space becomes rank-deficient by one, Jui and Sun [8] claimed that the manipulator can cross a singularity if its velocity and acceleration profile is suitably constrained. Ider [9,10] considered higher rank deficiencies of the combined coefficient matrix of the constraint and actuator forces, and derived the general conditions that the end-effector velocities and accelerations should satisfy at the singular positions. Briot and Arakelian studied further the physical interpretation of these criteria [11], and extended them to rigid-link flexible-joint [12] and flexible [13] parallel manipulators. Recently, Briot et al. [14] presented some additional experimental results and further extensions on them. In another recent study, Pagis et al. [15] proposed to simultaneously satisfy these conditions and some of their time derivatives for improving robustness to model uncertainties.

Basically, the aforementioned conditions correspond to making the dynamic equations consistent at the singular points, as also explained in Refs. [8–14]. There should be no doubt that consistency is a must for a physically realizable task. However, the present paper analytically proves that it is not sufficient alone for the boundedness of the inverse dynamic solution near singular positions. Hence, one of the main contributions of this study is the derivation of two complete sets of conditions for avoiding unbounded

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actuator forces while passing through singular configurations. Under these newly derived conditions, type II singularities of parallel robots are then shown to be removable from the inverse dynamics solution. All these are achieved by a novel analysis of this type of singularities based on the limit concept. Another originality of the paper is the proposal for the definitions of natural and actual orders of type II singularities.

Before finishing this section, readers may need to be reminded that type II singularities are also known in the literature as direct kinematic singularities [2,6], force singularities [8], or drive (or actuation) singularities [9,10]. One should also recall from calculus that a removable singularity of a function is a point at which the function is not defined, but its limit as that point is approached exists and is a finite value [16–19].

2. Notations and problem statement

Consider a non-redundant parallel robot with n degrees of freedom. There is at least one closed loop. The closed-chain system can be transformed into a virtual open-tree system by removing a sufficient number of passive joints. Denote by m the degree of freedom of the so-obtained open-tree system. The $m - n$ loop-closure constraint equations are then expressed as

$$\phi_i(\mathbf{q}) = 0, \quad i = 1, \cdots, m - n \tag{1}$$

where $\mathbf{q} = [q_1 \ \cdots \ q_m]^T$ is the vector of the joint variables of the open-tree system.

Let $\mathbf{q}^a = [q_1^a \ \cdots \ q_n^a]^T$ and $\mathbf{q}^p = [q_1^p \ \cdots \ q_{m-n}^p]^T$ denote the vectors of the joint variables of the active and passive joints, respectively. Then, the dynamic equations of the robot can be written as follows:

$$\tau_k = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k^a}\right) - \frac{\partial L}{\partial q_k^a} - \sum_{j=1}^{m-n} \lambda_j \frac{\partial \phi_j}{\partial q_k^a}, \quad k = 1, \cdots, n \tag{2}$$

$$\sum_{j=1}^{m-n} \lambda_j \frac{\partial \phi_j}{\partial q_i^p} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i^p}\right) - \frac{\partial L}{\partial q_i^p}, \quad i = 1, \cdots, m - n \tag{3}$$

where t is the time, $L = L(\mathbf{q}, \dot{\mathbf{q}})$ the Lagrangian, τ_k denotes the actuator force corresponding to q_k^a, and λ_j, $j = 1, \cdots, m - n$ represent the constraint forces. Recall that the Lagrangian is defined as

$$L = K - V \tag{4}$$

where $K = K(\mathbf{q}, \dot{\mathbf{q}})$ and $V = V(\mathbf{q})$ are the total kinetic and potential energies of the open-chain system, respectively.

Supposing that there is no type I singularity, one can find the required displacements, velocities and accelerations of the joints for a prescribed motion of the end-effector. Hence, the required actuator forces can be determined from Eqs. (2) as long as the constraint forces can be obtained through Eqs. (3). Before proceeding further, it is convenient to rewrite Eqs. (3) in matrix form:

$$\mathbf{A}\boldsymbol{\lambda} = \mathbf{R} \tag{5}$$

where $\boldsymbol{\lambda} = [\lambda_1 \ \cdots \ \lambda_{m-n}]^T$ is the unknown vector of the constraint forces, $\mathbf{A} = \mathbf{A}(\mathbf{q})$ is the associated coefficient matrix and $\mathbf{R} = \mathbf{R}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ is the right-hand side vector. The elements of $\mathbf{A}$ and $\mathbf{R}$ are given below:

$$A_{ij} = \frac{\partial \phi_j}{\partial q_i^p}, \quad i = 1, \cdots, m - n, \ j = 1, \cdots, m - n \tag{6}$$

$$R_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i^p}\right) - \frac{\partial L}{\partial q_i^p}, \ i = 1, \cdots, m - n \tag{7}$$

A type II singularity can be characterized with the rank deficiency of $\mathbf{A}$ [10], which, in general, results in unbounded constraint forces (and hence unbounded actuator forces) in its neighborhood. The following is a list of notations that are used throughout the rest of the paper:

- Denote the cofactor of the matrix element A_{ij} by C_{ij}.
- Let t_s be the time when $\mathbf{A}(t)$ becomes singular.
- Let ν denote the nullity of $\mathbf{A}(t_s)$, i.e. let $\mathbf{A}(t_s)$ be rank-deficient by ν.
- For any function $f(t)$, denote by $f^{(\omega)}(t)$ its ωth time derivative, the zeroth order derivative being the function itself.
- $\delta = \delta(t)$ is used to represent the determinant of $\mathbf{A}(t)$ (The well-known notations $|\mathbf{A}|$ and $\det \mathbf{A}$ are also interchangeably used in the numerical examples section).

2.1. Motivation and outline of the paper

As mentioned previously, the present paper aims to contribute to the existing literature by studying the limits of the inverse dynamics solutions of parallel robots as the singularity time is approached. The analysis can be started by rearranging Eq. (5) as

$$\boldsymbol{\lambda}(t) = (\mathbf{A}(t))^{-1}\mathbf{R}(t)$$