Mechanism and Machine Theory 107 (2017) 105–122

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

The dynamic coupling behaviour of a cylindrical geared rotor system subjected to gear eccentricities

Wennian Yu[a,*], Chris K. Mechefske[a], Markus Timusk[b]

[a] Department of Mechanical and Materials Engineering, Queen's University, Kingston, ON, Canada K7L 3N6
[b] Bharti School of Engineering, Laurentian University, Sudbury, ON, Canada P3E 2C6

ARTICLE INFO

Keywords:
Cylindrical geared rotor system
structure vector
eccentricities
dynamic coupling

ABSTRACT

A general dynamic model for the cylindrical geared rotor system with local tooth profile errors and global mounting errors is developed. This model includes a finite element (FE) model for the shaft structure, a lumped-parameter bearing model and a 3 dimensional (3D) gear mesh model based on previous work. Gear geometric eccentricities, unloaded static transmission error, as well as the fluctuation of the mesh stiffness are included in this model to study the dynamic coupling behaviour of the transverse and rotational motions of gears subjected to gear eccentricities. The simulation results are compared against the experimental results provided in the literature, which proves that the proposed modal is suitable to study the dynamic behaviour of a cylindrical geared rotor system. The influence of the helical angle, the number of teeth, gear profile error, and gear eccentricities on the dynamic coupling behaviour were studied. It was found that the dynamic coupling between the transverse and rotational motions of the gears becomes apparent in the low speed range when resonances are excited either by the gear profile error or mesh stiffness fluctuation. The analysis results of this research are useful for the dynamic analysis of gear transmission systems subjected to gear eccentricities.

1. Introduction

Gear transmission systems are widely used in many industry applications, such as automotive, wind turbines, mining, marine and industrial power transmissions. With the increasingly tight restrictions on the vibration and noise that may be generated from gear transmission systems, the prediction and control of gear vibration and noise are important considerations.

Currently, dynamic analysis of gear transmission systems remains an essential and important method to simulate gear dynamic behaviour [1–6]. It can help researchers to implement suitable solutions to reduce gear vibration and noise. Most early research focused on the modeling of a single spur gear pair supported by flexible shafts and bearings. Ozguven [7] and Wang [8] have carried out a comprehensive review of mathematical models used in this case, where gear torsional vibrations are the main concern. Although neglecting lateral vibrations might provide a good approximation for systems having shafts with small compliances, the dynamic coupling between the transverse and torsional vibrations due to the gear mesh affects the system behaviour considerably when the shafts have high compliances (i.e. geared rotor systems). In the literature, various formulations are used to represent the shaft flexibilities, ranging from simple equivalent lumped springs to finite elements. As a result, lumped mass and finite element analysis are both widely used to couple the lateral and torsional dynamics typical of geared rotor systems. In some other research, the transfer matrix method is also commonly used to study the flexural-torsional coupling behaviour [9–12]. However, with the

* Corresponding author.
E-mail address: wennian.yu@queensu.ca (W. Yu).

http://dx.doi.org/10.1016/j.mechmachtheory.2016.09.017
Received 1 April 2016; Received in revised form 8 August 2016; Accepted 18 September 2016
0094-114X/

growth of computing power, the finite element method has become dominant as the most efficient and accurate modeling method for the rotor dynamic studies. The dynamic investigations of a helical gear pair can be found in a variety of studies, where a 3D model was normally built to study the complex coupling amongst the transverse, torsional, axial and rotational motions of gears [13,14].

Gear geometric eccentricity is one of the most common gear mounting errors. Therefore, it should be an important consideration in the process of dynamic simulation. Research regarding it can be found in [9–14]. Kaharman et al. [10] developed a finite element model for spur geared rotor systems and analyzed the influence of geometric eccentricities, mass unbalances, static transmission error, and mesh stiffness variation on the response. Velex et al. [14] built a comprehensive 3D model for analyzing the influence of tooth shape deviations and mounting errors on gear dynamics. Their work forms the basis of the 3D gear model used in this research for the gear dynamic analysis with gear eccentric errors. Choi et al. [11] used the transfer matrix method to study the coupling motion of the lateral and torsional vibration of spur geared rotor systems with geometric eccentricity. Based on the previous work, Zhang et al. [12] proposed a dynamic model of a multi-shaft geared rotor system which consists of a finite element model of shaft structures and a helical gear model with gear eccentricity and static transmission error. The steady-state responses due to these two excitations were studied. In all of the above studies, the inertial force due to gear eccentricity is simplified as an uncoupled standard centrifugal inertial force, whereas the dynamic coupling effect on the inertial force due to gear torsional vibrations is normally ignored.

Therefore, the objective of this study is mainly to investigate the role of the coupling terms in the gear eccentricity induced inertial force, and analyze their influence on the dynamic behaviour of the cylindrical geared rotor system. In order to achieve this, a general dynamic model for the helical geared rotor system with local tooth profile errors and global mounting errors was developed. This model includes a finite element (FE) model for the shaft structure based on the work in [10–12], a lumped-parameter bearing model and a 3D gear mesh model mainly based on Velex's work [14]. This combined model was verified firstly by comparing natural frequencies of a one-stage spur geared rotor system against those presented in the literature, and secondly by comparing the simulated dynamic transmission error of a helical gear pair with previous experimental results. Furthermore, the dynamic coupling effect in the gear eccentricity induced inertial force on the gear dynamic behaviour of a helical geared rotor system is intensively investigated, and some conclusions are drawn.

2. Dynamic model

A typical geared rotor system consists of the following elements: (1) shafts, (2) rigid disks, (3) flexible bearings, and (4) gears. When two shafts are not coupled, each gear can be modeled as a rigid disk. However, when they are in mesh, these rigid disks are connected by a spring-damper element representing the mesh stiffness and damping.

2.1. Shaft model

In the literature, the finite element formulation is widely used to model shafts including either Euler beam models [10] or Timoshenko beam models [12,15]. Here, a Timoshenko beam formulation is employed as it can include the effects of the translational and rotary inertia, gyroscopic moments, and the shear deformation, which are expected to be significant especially in high-speed cases.

There are 2 nodes for each Timoshenko beam element (as shown in Fig. 1), and six degrees of freedom at each node:

$$\boldsymbol{u}^{\boldsymbol{e}}=\{x_A,\ y_A,\ z_A,\ \theta_{xA},\ \theta_{yA},\ \theta_{zA},\ x_B,\ y_B,\ z_B,\ \theta_{xB},\ \theta_{yB},\ \theta_{zB}\}^{\mathrm{T}} \tag{1}$$

The mass $\boldsymbol{M}_{si}^{e}$, stiffness $\boldsymbol{K}_{si}^{e}$ and gyroscopic moment $\boldsymbol{G}_{si}^{e}$ matrices for the ith finite shaft element can be found in [12,15,16], and will not be detailed here. These matrices are assembled to form the mass $\boldsymbol{M}_s^j$, stiffness $\boldsymbol{K}_s^j$ and gyroscopic moment $\boldsymbol{G}_s^j$ matrices for the jth shaft (j=1 to N where N is the number of the shafts considered; i=1 to m_j where m_j is the number of finite elements used to define the jth shaft). Therefore, the overall shaft mass $\boldsymbol{M_S}$, stiffness $\boldsymbol{K_S}$ and gyroscopic moments $\boldsymbol{G_S}$ can be expressed as:

$$\boldsymbol{M}_s=\begin{bmatrix}\boldsymbol{M}_s^1 & & & \\ & \boldsymbol{M}_s^2 & & \\ & & \ddots & \\ & & & \boldsymbol{M}_s^N\end{bmatrix},\ \boldsymbol{K}_s=\begin{bmatrix}\boldsymbol{K}_s^1 & & & \\ & \boldsymbol{K}_s^2 & & \\ & & \ddots & \\ & & & \boldsymbol{K}_s^N\end{bmatrix},\ \boldsymbol{G}_s=\begin{bmatrix}\boldsymbol{G}_s^1 & & & \\ & \boldsymbol{G}_s^2 & & \\ & & \ddots & \\ & & & \boldsymbol{G}_s^N\end{bmatrix} \tag{2 - a, b, c}$$

This results in symmetric, square matrices of dimension q where $q = 6\sum_{j=1}^{N}(m_j+1)$ and is the total number of degrees of freedom of the system at hand.

2.2. Bearing model

Typically, each shaft is supported by at least two rolling element bearings of varying types and parameters. Normally, they are modeled as stiffness only and the cross terms and damping are ignored [12,16], i.e. six spring stiffnesses in the x, y, z, θ_x, θ_y, θ_z directions: