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Effect of mesh stiffness of healthy and cracked gear tooth on modal and frequency response characteristics of geared rotor system

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ABSTRACT

Geared rotor systems are used in industrial machinery, automotive applications etc. to provide rotational speed changes and torque variations by transmitting rotational motion from driving shaft to the driven shaft. Gear tooth contact, as modelled using gear mesh stiffness and damping, considerably affects the dynamic characteristics of the system. This work primarily studies the effect of mesh stiffness and damping due to gear pair on the modal characteristics of the geared flexible rotor-shaft system supported on compliant bearing. The rotor-shaft is modelled using Timoshenko beam elements; whereas non-proportional viscous damping model is used to represent bearing and gear mesh damping. Fatigue loading causes development of gear tooth crack, which results in reduction of mesh stiffness. The effect of various cases of cracked gear tooth has also been investigated on the modal characteristics and frequency response functions of the system. The changes in the modal and frequency response characteristics due to cracked gear tooth have been compared with that of healthy geared rotor system. The study may prove helpful in detection of faults developing in the gear tooth by observing the changes in the dynamic characteristics of the system.

1. Introduction

Geared devices are used for power transmission ranging from industrial applications to everyday household appliances. Power is transmitted due to direct contact of the teeth on the driving shaft and the driven shaft. It results in coupling of lateral and torsional degrees of freedom at the gear and pinion. This work primarily studies the dynamic behaviour of coupled geared rotor-shaft system supported on compliant bearings. The coupling resulting from direct contact of gear and pinion alters the dynamic characteristics of the geared rotor system. Dynamic characteristics of a system include natural frequencies, damping factors, mode shapes and frequency response functions [1]. Modal analysis is a tool to find out the dynamic characteristics of the structures. Frequency response function (FRF) gives a measure of magnitude and phase of the response as a function of frequency at any point on a structure subjected to unit harmonic force at the same point (referred to as point FRF) or some other point (referred to as cross FRF) on the structure.

Modal analysis is classified into two types namely, (i) experimental modal analysis (EMA) and (ii) theoretical/numerical modal analysis. Experimental modal analysis uses measurement of frequency response functions and then estimates modal characteristics from the measured data. It requires measurement of one complete row or column of the FRF matrix for complete characterization of self-adjoint systems, and this process is referred to as modal testing [2]. Finite element analysis is widely used for numerical modal

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analysis [3,4]. However, the accuracy of finite element model should be checked before relying on the estimated results using finite element analysis. The development of updated finite element model requires correct selection of various inertia, stiffness and damping parameters based on engineering judgment of measured data [5,6]. In case of geared rotor systems, it is important to properly model the gear mesh stiffness among other parameters. In literature, there are many studies on use of finite element model for dynamic behaviour prediction, fault identification and dynamic design of machinery structures [7–9]. However, such studies are very limited for geared rotor systems. In view of this, the present work primarily addresses the effect of mesh stiffness on the modal characteristics of geared rotor system using finite element method.

Mesh stiffness due to gear pair contact is an important parameter which influences dynamic characteristics as well as rotor response of the system. Therefore, correct estimation of the mesh stiffness is very crucial. In literature, mathematical basis for calculating the mesh stiffness is reported by many researchers. Yang et al. [10] proposed a value of Hertzian energy which is further extended by Yang et al. [11] to calculate time varying mesh stiffness (TVMS) of a gear pair using potential energy method by including bending energy and axial compressive energy with Hertzian energy. Various types of faults like crack, spalling, pitting, scoring, wear etc. may develop in the gear teeth over the time. These faults in the gear tooth have the effect of reducing the mesh stiffness of the gear pair and the extent of reduction is generally related to the severity of tooth damage. The reduction of mesh stiffness due to different gear tooth faults has been studied by many researchers using various methods. Pandya et al. [12] have adopted principle of linear elastic fracture mechanics (LEFM) to carry out crack propagation path studies with different contact ratio and predicted the change in TVMS for different crack propagation path. Pandya et al. [13] proposed cumulative reduction index (CRI), using a variable crack intersection angle, to represent the percentage reduction of total mesh stiffness and studied the effect of different gear parameters e.g. pressure angle, fillet radius and back-up ratio on the total TVMS. Saxena et al. [14] studied the effect of shaft misalignment and friction on total effective mesh stiffness for spur gear pair and showed that the misalignment and friction affect TVMS of the gear pair considerably. Further, Saxena et al. [15] studied the effect of different spall shapes, size and location considering sliding friction on TVMS of the gear pair. Chen and Shao [16] proposed an analytical mesh stiffness model which studies the effect of the gear tooth errors like tooth profile modifications, applied torque and gear tooth root crack on the mesh stiffness of spur gear pair.

All these studies describe ways for calculation of mesh stiffness of healthy geared rotor and investigate the effect of gear tooth fault on reduction of the mesh stiffness. However, these studies did not report the influence of mesh stiffness on modal characteristics and rotor response. The variation in mesh stiffness directly influences the dynamic characteristics of the geared rotor system. Iwatsubo et al. [17] studied coupled lateral-torsional vibration of geared rotor using transfer matrix method by assuming tooth rigidity as a linear spring. Wu et al. [18] studied the effect of crack growth in gear tooth on total mesh stiffness and vibration response. Choy et al. [19] analysed multi stage gear transmission system by considering it as an enclosed structure and simulated transient and steady state vibrations due to torque variation, speed change, rotor imbalances and gear box support motion excitations. Vinayak et al. [20] studied multi-mesh transmission with external fixed centre helical and spur gears using dynamic modelling by using simplified model of shaft-bearing subsystem and proposed a computationally efficient multi-mesh geared system. Li et al. [21] studied coupled lateral – torsional dynamics of rotor bearing systems of spur bevel gear and concluded that critical speeds of coupled modes are different from those of uncoupled modes. Pedersen et al. [22] reported the mathematical foundations of time-variant modal analysis including the effect of TVMS on spur gear dynamics. Driot et al. [23] discussed variability of mesh stiffness and modal dynamic behaviour of gear pair system due to manufacturing errors in gear tooth; however, their study was focused on gear pair only and did not consider shaft and bearing flexibility into account. The modal analysis studies involving various elements of the geared rotor systems (i.e. flexible rotor shaft, mesh stiffness and damping, bearing stiffness and damping) to find out the effect of mesh stiffness on modal behaviour are found to be very limited in the literature. Such studies can be very helpful in gear fault diagnosis as gear tooth fault directly influences the mesh stiffness and in turn the modal data of the geared rotor system, which can be reflected in the results of measured frequency response functions and forced response of the system. Kahraman et al. [24] developed a finite element (FE) model of a geared rotor system supported on flexible bearing considering rotary inertia of shaft, axial loading on shaft, flexibility and damping of bearing, stiffness and damping of gear mesh and concluded that bearing compliances can greatly affect the dynamics of geared system. However, the effect of gear tooth fault was not considered in their work. Mohammed et al. [25] carried out dynamic response and time-frequency analysis for gear tooth crack detection. However, the authors used a 6 DOF dynamic gear model and did not include detailed modal analysis of the geared rotor system to see the effect of mesh stiffness and tooth crack on the mode shapes. Finite element method provides an efficient tool to model various elements of the rotor systems including shaft flexibility, bearing and mesh stiffness as well as damping and way to perform detailed analysis for modal frequencies, mode shapes and FRFs. Therefore, this work uses finite element method for the modal analysis of geared rotor system. The work attempts firstly the modal analysis of a geared rotor system to investigate the effect of mesh stiffness on natural frequencies and mode shapes of the system. This is followed by modal analysis study for a cracked gear rotor system by considering TVMS to find out the effect of cracked gear tooth on the natural frequencies and the FRFs.

2. Modal analysis of geared rotor system

Modal analysis of rotating machinery involves estimation of natural frequencies, modal damping factors, mode shapes, critical speeds, frequency response functions etc. Many studies have been found in the literature about modal analysis of rotors; however most of these studies are limited to modal analysis of rotors supported on single rotor-shaft. A geared rotor system includes two shafts i.e. driving shaft, driven shaft apart from other elements like bearing supporting the shafts, gear and pinion etc. Hence, finite element model of geared rotor system includes modelling of all these elements, which can further be used for the modal analysis.