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Modular kinematics and statics modeling for precision positioning stage

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ABSTRACT

Compliant amplifying mechanisms and guiding mechanisms are prevalently adopted in most present piezo-actuated precision positioning stages. By considering the coupled relations among piezoelectric actuator, compliant amplifying mechanism and guiding mechanism, an analytical method towards the statics modeling of the kinematic performance for precision positioning stages is presented. In this method, these three parts are separately modeled and coupled together in series by defining input/output stiffness. Then two static displacement models with different stiffness definitions are established and mathematically proved to be equivalent, both of which may be versatile for practical applications. A comparison of the proposed method with the finite element analysis for a flexure-based XY positioning stage reveals less than 10% deviations and indicates that the workspace significantly decreases with consideration of the finite stiffness of piezoelectric actuator and the elastic effect of guiding mechanism. The proposed method offers a modular and assembled statics modeling tool for analysis and design of a wide class of flexure-based precision positioning stages.

1. Introduction

Due to high resolution, large output force and high frequency response, piezoelectric actuators are frequently utilized to drive a majority of the state-of-art precision positioning stages [1,2]. Meanwhile, the limited output displacement of piezoelectric actuators is often transferred and magnified by various flexure hinge-based compliant mechanisms [3,4]. To restrict parasitic motions and to avoid undesired cross-coupling for performing only desired motions, guiding mechanisms are also often introduced at the output end of a compliant amplifying mechanism, especially for multiple degree-of-freedom (DOF) precision positioning stages [5].

Fig. 1 provides several specific piezo-actuated precision positioning stages with displacement amplifying and guiding mechanisms [6,7]. The function of guiding mechanisms can be generalized as: a) To minimize the parasitic motion and undesired vibration mode in the output direction (e.g. the first mode of rocking side-to-side in Fig. 1(b) is improved by introducing two flexible beams); b) To avoid cross-coupling for independent multi-DOF output displacement, as shown in Fig. 1(c). Actually, many present serial/parallel-kinematic precision positioning stages are designed with motion guiding mechanisms for enhancing response bandwidth and better control accuracy. Kenton et al. [8] developed a high-bandwidth three-axis nano-positioning stage in which serial double-compliant beams were employed for guiding the displacement of the platform to minimize the parasitic motion. Polit et al. [9] presented a high-bandwidth parallel-kinematic *XY* nano-positioner by using a doubly clamped beam and parallel compliant beams to decouple the motions in the *X*- and *Y*-directions. Yong et al. [10,11] designed a piezo-actuated *XY* nano-positioning stage

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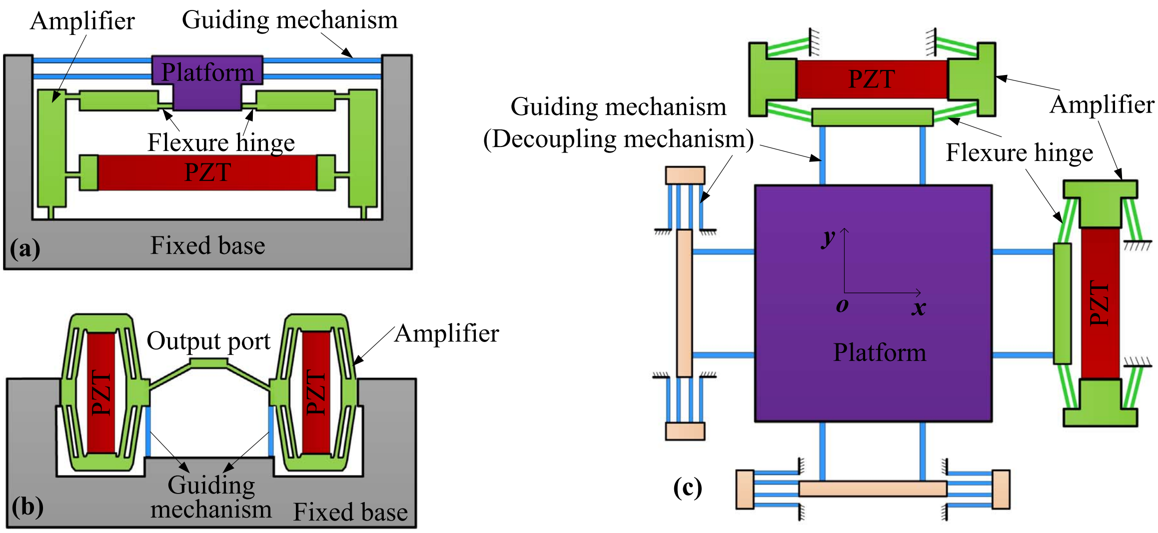

Fig. 1. Typical precision positioning stages. (a) Single-DOF stage [6] (Input decoupling structure is not included here), (b) Multistage-based *XY* stage, (c) Parallel *XY* stage [7].

by using four pairs of flexible beams to weaken the cross-coupling between the two axes. Li et al. [12] developed an *XY* precision positioning stage featuring with decoupled actuation and decoupled output motion by employing a compact displacement amplifier and compound parallelogram flexure. Many other similar configurations can also be seen in [13–16] for details.

From the viewpoints of achieving high-bandwidth and high-accuracy control for precision positioning stages, guiding mechanisms do work. Therefore, stages with guiding mechanisms have a good robustness against external disturbance and internal cross-coupling effect. However, output displacement is bound to reduce compared with the case having no guiding mechanisms; design and analysis procedures also become complicated. To describe and approximate the static and dynamic performance of these precision positioning stages, many analytical methods have been proposed in the last few decades. An inverse kinematic model similar to the finite element method (FEM) was first proposed by Ryu et al. [17] and has been widely utilized for the static and dynamic modeling of multi-DOF nano-positioners [14,18]. Based on the well-known pseudo-rigid-body model (PRBM) [19], many analytical modeling methods were also developed for kinematic and static analysis of nano-positioners by combining the matrix method [12,20,21], elastic beam theory [22,23] and Castigliano's second theorem [6,24]. These modeling methods, however, seldom address the quantitative issue of attenuated output displacement due to the influence of the finite stiffness of piezoelectric actuators and the elastic effect of guiding mechanisms.

Differing from the previous methods, the modular and assembled modeling strategy is introduced in this paper to propose two mathematically equivalent static displacement models for piezo-actuated and flexure-based precision positioning stages. In the method, piezoelectric actuator, compliant amplifying and guiding mechanisms are separately considered and modeled, then coupled together in series by defining port stiffness. The kinematics of a precision positioning stage can be easily quantified only by calculating the displacement amplification ratio and the input/output stiffness of compliant amplifying mechanism as well as the equivalent stiffness of guiding mechanism defined in the proposed models. The advantage of the method is that an attenuated output displacement closer to the actual design can be accurately predicted by considering the finite stiffness of piezoelectric actuators and the elastic effect of guiding mechanisms. Besides, the displacement amplification ratio and the port stiffness of some typical compliant amplifying and guiding mechanisms have been modeled based on PRBM in the previous research and can be directly assembled into the proposed models.

The rest of this paper is organized as follows: theoretical modeling is deeply conducted in Section 2; applying the presented method to a typical rhombus-type amplifier-based *XY* positioning stage is reported in Section 3, where issues of model verification and simplification are also investigated; lastly, concluding remarks are given in Section 4.

2. Theoretical modeling

Configuration of many present piezo-actuated, flexure-based precision positioning stages can be abstracted to be a serial two-port mechanical network, as shown in Fig. 2. Considering the finite stiffness of the piezoelectric actuator, a reduced displacement will be generated with a force caused by the reaction to the compliant amplifying mechanism and the guiding mechanism when a voltage is applied to the piezoelectric actuator. The attenuated output displacement of the piezoelectric actuator, which is also the input displacement of the compliant amplifying mechanism, is first transmitted or amplified by the compliant amplifying mechanism and then reduced again due to the elastic impedance of the guiding mechanism. Taking the compliant amplifying mechanism as the study object, its quantitative input-output static displacement and force relations have the analogous form of the generalized Hooke's law as follows: