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Clearance-induced output position uncertainty of planar linkages with revolute and prismatic joints

Kwun-Lon Ting[a,*], Kuan-Lun Hsu[b], Zetao Yu[b], Jun Wang[c]

[a] Center for Manufacturing Research, Tennessee Technological University, Cookeville, TN, USA
[b] Tennessee Technological University, Cookeville, TN, USA
[c] Hubei University of Technology, Wuhan, Hubei, China

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ABSTRACT

The paper presents the first kinematic model to quantify the effects of the joint clearance stack up on the nominal output link position of linkages containing revolute and prismatic joints. It demonstrates the modeling of each joint clearance, how the clearance stacks up is formed, and how the position uncertainty is affected by each joint clearance as well as the combination of all joint clearances. The novelty lies on the simplicity of the kinematic model resulted from the use of Ting's rotatability laws. For the worst case scenario, a chaotic clearance induced output uncertainty analysis at a nominal position of an N-bar linkage is reduced to a simple four-bar linkage analysis. The treatment is valid for all single loop N-bar linkages containing revolute and prismatic joints. The proposed kinematic model is showcased with a slider crank mechanism and an inverted slider crank mechanism.

1. Introduction

For a linkage or manipulator containing rigid links, position error may be caused by link tolerance as well as joint clearance. Joint clearance is the source of position uncertainty. However, joint clearance is often needed to permit assembling or relative motion between the connected links. Joint clearance may also be caused by wear.

The effects of joint clearance of mechanisms have attracted many studies with different models. Most of the models on joint clearance can be divided into two categories, the model of clearance for revolute joints [1] and prismatic joints [6,9–16,21,22]. The treatment is usually associated with kinematics [2–6], dynamics [7–20], tribology [21,22], probability [23–25], and elasticity [26–29], or their combinations. Because of the complexity or even chaos, complex and tedious mathematical treatment, numerical method or even genetic algorithm is a norm [30–34].

For most clearance related researches, a revolute joint clearance is commonly characterized as a massless clearance link with a redundant degree of freedom at the connection between two rigid links [1]. Several approaches were based on the clearance model for revolute joints. Grant and Fawcett [7] considered the impact of a revolute clearance joint at the coupler-rocker bearing of a four-bar linkage theoretically and experimentally. Tsai and Lai employed the screws theory for evaluating the transmission performance of linkages [2] and determining the instantaneous configurations [3] of linkages with joint clearances. In addition, both probabilistic [23–25] and non-probabilistic method [27–29] were also often adopted to consider the uncertainty performance of mechanisms and manipulators. Erkaya and Uzmay [30–34] extensively used the genetic algorithm to study different kinds of planar mechanisms with

* Corresponding author.
E-mail addresses: kting@tntech.edu (K.-L. Ting), kaunlwun.shu@gmail.com (K.-L. Hsu), zyu43@students.tntech.edu (Z. Yu), junwang@mail.hbut.edu.cn (J. Wang).

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revolute joint clearances. To more precisely characterize the clearance-induced effect, the link deformation was taken into consideration when the penetration of each joint occurred [17]. Yaqubi et al. [18] provided a control scheme for maintaining continuous contact in joints to prevent undesired effects such as extra noise and shorter lifetime. Pereira et al. [19] presented a generalized geometry to describe the contact conditions for the roller chain drives with the revolute clearance joints. Elastic foundation model was proposed by Li et al. [26] to consider the impact of flexibility on the dynamic response of a planar slider-crank mechanism. Due to the advancements in computer and software technology, commercial packages such as RecurDyn [20] can be used to validate existing investigations regarding dynamics of a slider-crank mechanism under the influence of imperfect revolute joints. However, position uncertainty is rooted in geometry. It is a kinematic issue fundamentally and yet very few literatures [4] attempt to establish the kinematical model to explain and predict the position uncertainty.

Several different approaches have also been proposed to investigate the effect of prismatic joint clearances. Wilson and Fawcett [9] predicted the dynamic motion of a slider-crank linkage with an imperfect prismatic joint by deriving the equations of motion for all possible orientations of the slider relative to guide surface. Farahanchi and Shaw [10] tried to characterize the chaotic and periodic dynamic response of a slider-crank mechanism with a clearance at a prismatic joint. Chen [6] conducted a forward kinematic analysis for estimating the accuracy of a slider-crank linkage with clearance-affected joints. Flores et al. [11,22] demonstrated the clearance-induced nonlinearity by introducing the contact-impact forces for modelling the collision between the slider and guideway. For the existing models of prismatic clearance joints, the slider was assumed to have a finite clearance gap between the guideway. Due to the complexity of the contact phenomenon between the slider and the guideway, the model usually leads to complex mathematical derivation. The complexity is further compounded by the consideration of dynamic and elastic behaviors [12–16,21].

This paper recognizes that a clearance-induced position uncertainty is fundamentally a kinematic issue [4]. A kinematic model should be established to explain and understand the basic geometry and formation of clearance-induced position uncertainty. A question is how to model a prismatic joint clearance. An even bigger problem is that joint clearances introduce high level of redundant degrees of freedom. When all joint clearances are taken into consideration, the system becomes chaotic. How can one explain the formation of position uncertainty and predict the effect of an individual joint clearance or the combination of all joint clearances? What if the clearances of revolute and prismatic joints are present? These are issues to be addressed in this paper. The result is made possible with Ting's N-bar rotatability laws [35–37] and a new model for a prismatic joint clearance.

2. Models of joint clearance for revolute and prismatic pair

For a planar linkage with the lower pairs, there may exist two kinds of joint clearances, which are the revolute and prismatic joint clearance, respectively. For a revolute joint clearance as shown in Fig. 1(a), it can be characterized as the diametric difference of a pin and hole at the joint. It is assumed throughout all research that the pin is in constant contact with the hole of the joint and no penetration occurs at each joint. Hence, the distance between the center of the pin and that of the hole remains constant. The coupled links, which are adjoined by a revolute clearance joint, are equivalently connected by a virtual clearance link. The length of this virtual clearance link δ_R is equal to the difference between the radius of the hole and that of the pin, which is also equal to one half of revolute joint clearance.

In the case of a slider shown in Fig. 1(b), it can be regarded as a rotating link where the center point of a fixed revolute joint is at infinite. In other words, the link has an infinite length and the orientation of the link is always perpendicular to the sliding path of the slider. Therefore, if there exists a clearance at a prismatic joint, the slider can be characterized by an infinitely long link with a clearance link δ_P. The clearance link δ_P physically represents the positional deviation of the slider measured from the exact sliding path which is depicted in center line. The positional deviation δ_P can be located possibly either above or below the exact sliding path. The length of a clearance link δ_P is determined according to a slider configuration relative to guide surfaces and will be further explained below.

The magnitude of a joint clearance has a direct influence on the performance of a planar linkage. In fact, the length of a clearance link is a variable for both of the revolute joint and prismatic joint. By assuming that a constant contact exists between the pin and hole at a revolute joint, the clearance link δ_R has a constant length equal to one half of the joint clearance. On the other hand, the

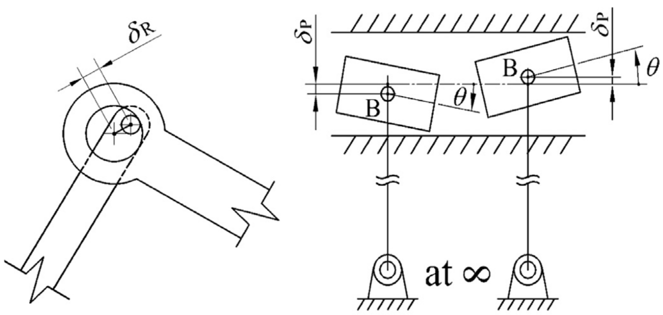

Fig. 1. Models of joint clearance for the (a) revolute pair and (b) prismatic pair.