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Mathematical definition and computerized modeling of spherical involute and octoidal bevel gears generated by crown gear

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ABSTRACT

The involute profile is the most commonly used tooth profile for cylindrical gears. However, its counterpart for bevel gears is not commonly used because its generation is not compatible with the use of easy-to-manufacture cutting tools and market-oriented generation methods. Nowadays, with additive manufacturing technology growing across industries to manufacture parts and full-scale products, the spherical involute profile deserves to be given the possibility to become the reference profile for bevel gears generated by additive manufacturing, forging or plastic molding. In this work, both the spherical involute and the octoidal form systems for bevel gears are mathematically defined. The geometry of their respective crown-gears for generation is derived and the computerized modeling of the geometries of the generated bevel gears presented. The comparison between the obtained gear tooth geometries is performed from the point of view of the obtained geometric deviations, their behavior to the change of the shaft angle and the evolution of contact and bending stresses for different values of torque applied to the pinion. The results showed that the spherical involute profile can handle better with the changes in the shaft angle, although the octoidal profile presents lower contact and bending stresses.

1. Introduction

Straight-tooth bevel gears are the simplest type of bevel gears that can be used for power transmission between intersecting shafts. They are commonly referred to as straight bevel gears for brevity. Shafts for power transmission with straight bevel gears are typically mounted at a shaft angle of 90°, but can be designed to work with different shaft angles. They are widely applied in low-speed applications or static loading conditions [1]. The most traditional application of straight bevel gears is in differential drives, in which the speed is low and the load type is mainly static.

The involute profile is the most commonly used tooth profile for cylindrical gears, because it has an extensive list of advantages including that tools for its generation can be produced with high precision. However, the same cannot be said for bevel gears where there is no a commonly used reference profile. In this work, the mathematical definition of both

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spherical involute and octoidal profiles will be presented and the computerized generation by their respective crown gears derived.

Many research studies have been dealing with the definition of the spherical involute profile and its application for bevel gears [1–8]. All of them have contributed in one way or another to the further understanding of the spherical involute profile and the configuration of the comprehensive definition provided here. Further application of spherical involute tooth profiles for spiral bevel gears whose generating curve is an arc of a circle is presented in [9]. Also, in [10], a new methodology to provide gear designers with the ability to mathematically specify two gear teeth independent of each other and ensure perfect motion transmission when engaged in mesh is proposed.

The main drawback for the application of the involute profile in straight bevel gears has been that there is no an effective process of manufacturing because its generation is not compatible with the use of easy-to-manufacture cutting tools. However, nowadays with additive manufacturing technology growing across industries not just for producing prototypes but to manufacture parts and full-scale products, the spherical involute profile deserves to be given the possibility to become the reference profile for bevel gears generated by additive manufacturing, forging or plastic molding.

When deriving the spherical involute profile, the definition of the octoidal profile is clearly seen as a way to simplify the definition of the spherical involute profile and will be presented as well in this paper. The octoidal tooth form is exclusive of generated bevel gears. There is no analogous profile for cylindrical gears. It is obtained when the crown gear that theoretically generates the bevel gear have planar flanks. The first written reference that have been found of the octoidal profile, dating from 1899, appears in the American Machinist, wherein an illustrated explanation of the term octoid and its difference from the involute system was provided. An extensive and thorough analysis on the geometric characteristics on the octoidal gear tooth system has been done in [3]. Recently, the same authors have presented a spatial version of octoidal gears via the generalized Camus Theorem [11].

In this paper, the computerized generation of both spherical involute bevel gears and octoidal bevel gears generated by crown gears is proposed and the obtained geometries compared in terms of deviations, results of tooth contact analysis and maximum level of contact and bending stresses. Although the geometry of spherical involute bevel gears can be obtained directly from the mathematical definition, the use of crown gears will allow further benefits as the possibility to incorporate geometry modifications on the generated surfaces by changing the geometry of the crown gear or even compensating errors of alignment caused by shaft deflections, although this topic is out of the scope of this paper.

2. Determination of pitch cones angles

The concept of pitch cones in bevel gears is analogous to the concept of pitch cylinders in spur and helical gears [12]. Fig. 1 shows the transformation of motion between two intersected axes z_1 and z_2 forming an angle Σ. The instantaneous axis of rotation OI is the line of action of the angular relative velocity $\boldsymbol{\omega}^{(12)}$ in relative motion of gear 1 with respect to 2 [13]. Here,

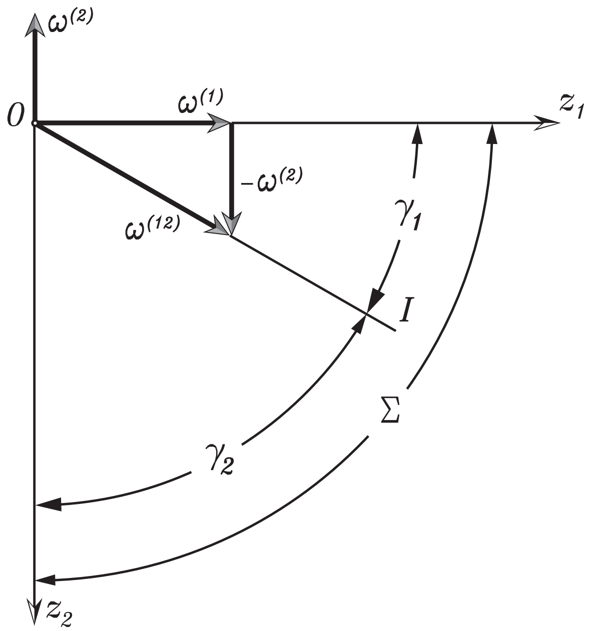

Fig. 1. Towards determination of pitch cone angles in bevel gears.