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Research paper

An automatic method for the connectivity calculation in planar closed kinematic chains

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ABSTRACT

In the initial stage of the creative design of some specific mechanisms such as the parallel manipulator, the connectivity between the base and the end-effector must satisfy the requirement. This paper proposes a new method for the connectivity calculation between two links, based on which an automatic method to acquire the connectivity matrix for planar closed kinematic chains is developed with the aid of C++ programming language. The method has several advantageous features resulting in lower complexity relative to the methods presented in the literature. The whole process to compute connectivity matrix using Floyd-Warshall algorithm, depth-first search (DFS) algorithm and the subchain mobility superposition algorithm is presented and the planar kinematic chains with up to 6 independent loops, together with the ones found in literature, are illustrated to show the validity and efficiency of the method. This method can build a convenient bridge to automatically generate the mechanisms, whose base and end-effector have the required connectivity, from the synthesized kinematic chains.

1. Introduction

During the mechanical conceptual design stage, the innovative design of mechanisms or robots traditionally relies on designers' knowledge, experience and intuition. However, this specific approach is lack of systematicness and reliability, and hardly leads to the optimum design [1–3]. In order to alleviate this issue, two procedures have been developed. The first procedure is to generate all the kinematic structures (kinematic chains) of the feasible mechanisms, and the second procedure is to select the proper mechanisms satisfying the required design constraints from the synthesized kinematic chains.

The first procedure is well-known as number synthesis of kinematic chains with a specified number of links and degrees of freedom (DOFs), and numerous structure synthesis methods have been proposed resulting in successful generation of various kinds of kinematic chains. Butcher [4] presented a hierarchical representation to enumerate 1-DOF kinematic chains with up to 14 links. Lee and Yoon [5] and Tuttle [6] used finite symmetry group to generate kinematic chains respectively. Sunkari and Schmidt [7] generated 1–4 DOF kinematic chains based on Mckay-type algorithm. Simoni [8,9] used group theory to enumerate 1–6 DOF kinematic chains up to 4 basic loops. Pozhbelko [10] introduced the structure theory to synthesize the open-, closed-, and mixed-loop kinematic chains. Yan [11–13] improved an approach based on graph theory to construct various atlases of generalized kinematic chains up to 16 links. Ding [14–18] established the complete set of planar non-fractionated kinematic chains with up to 6 independent loops and all possible degrees of freedom. Besides, the automatic methods for the sketching of kinematic chains,

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proposed one after another, give the designers a visualized graph representation to improve the visualization and understanding of the structures of mechanisms [19–25].

For the second procedure, the generated kinematic chains are classified according to the structural parameters and then are selected based on various mechanical design requirements [26–31]. However, the generated kinematic chains, especially for the chains with complex structures, are hardly applied to the mechanical creative design since the number of the kinematic chains is often so large that designers are almost impossible to manually analyze the structural characteristics of each kinematic chain. So the required design constraints, including the numbers of links and joints, the mobility (or degree of freedom i.e. DOF) of the mechanism, the type of joints and so on, are researched in order to select the synthesized kinematic chains according to the design requirements. For some specific mechanisms such as the parallel manipulators or robots, another important parameter, namely the connectivity between the base and the end-effector, should also be taken into consideration. The subject of investigating the connectivity in kinematic chains or mechanisms has received much attention in the literature.

The definition of the concept of connectivity was proposed by Hunt [32], that is, the connectivity between two specific links of a kinematic chain is the relative mobility between them. Phillips [33] introduced the term "joint in the bag equivalence" and assumed that all the links and joints between links *i* and *j* were hidden inside a flexible black bag. And the bag was regarded as an equivalent joint whose mobility was considered to be equal to the connectivity between links *i* and *j*. Tischler et al. [34–36] introduced the concept of variety to compute the connectivity in a given kinematic chain. A proposition and two corollaries were presented to determine the lower bounds for the connectivity, in order to determine the exact value of the connectivity between links *i* and *j*, one need to identify the lowest mobility subchain of the kinematic chain and check the position of links *i* and *j* relative to the corresponding subchain [35]. Shoham and Roth [37] utilized graph theory and the method with adding virtual edges to solve the problem of connectivity calculation. As a result, the connectivity calculation in closed mechanisms with one or two independent loops was discussed, and for a given open mechanism, the connectivity between links *i* and *j* was measured as the weighted distance between the corresponding vertices *i* and *j* in the graph representation of the mechanism. Tsai [28] assumed that each limb of a parallel manipulator was made up of an open chain and calculated the connectivity of a limb as the number of degrees of freedom associated with all the joints in the limb. Obviously, the method is equivalent to the one presented in [37] where mechanisms were represented by their graph representations. Based on Shoham and Roth's work [37], Belfiore and Benedetto [38] further studied the method with adding virtual edges and presented an algorithm for the numerical evaluation of connectivity in spatial kinematic chains with total and fractional mobility. However, some problems may arise when the algorithm deals with kinematic chains with partial mobility. Liberati and Belfiore [39] developed an automatic algorithm, called "circuits gradual freezing", to study the connectivity in planar and spatial kinematic chains. Besides the kinematic chains with total and fractional mobility, those with partial mobility were taken into consideration. Martins and Carboni [40] proved the proposition and corollaries proposed by Tischler [35] and presented an algorithm for connectivity calculation which reduced the complexity and enhanced the validity relative to the previous algorithms. However, the algorithm may confront difficulty when processes kinematic chains with a large number of independent loops, as described in the end of ref. [40]: "the algorithm here proposed is a valid solution for kinematic chains with a small number of independent loops, otherwise the number of subchains may increase dramatically, and the computational time required to perform the analysis may be unacceptably long". Based on the algorithm for connectivity calculation in [40], Simoni et al. [41–43] enumerated kinematic structures of parallel manipulators originated from a given kinematic chain with the application of connectivity and symmetry.

Since the connectivity in open kinematic chains is easy to be analyzed [28,37], the studies in the literature mainly focused on closed kinematic chains, including planar and spatial chains. In this paper, a new method for the calculation of connectivity in planar closed kinematic chains is proposed. Besides, to enhance the efficiency of the computation, the method in this paper is realized automatically with the aid of C++ programming language.

The overall structure of this paper is arranged as follows. In Section 2, the concepts of the kinematic chain, topological graph and adjacency matrix are introduced. In Section 3, a new formula for the calculation of the connectivity between two links is proposed, based on which a systematic method to acquire the connectivity matrix for planar closed kinematic chains is developed. And the advantageous features relative to the methods in literature are analyzed. In Section 4, the detailed process to compute connectivity matrix using Floyd-Warshall algorithm, DFS algorithm and subchain mobility superposition algorithm is described to show how the method is realized automatically with the aid of C++ programming language. In Section 5, the topological graph of a planar 12-link 3-DOF kinematic chain is taken as a calculation example to show the process of the connectivity calculation. The software interfaces of computing the connectivity matrix for several kinematic chains with up 6 independent loops are provided to show the efficiency of the method.

2. Basic concepts

In the process of conceptual design of mechanisms, the kinematic structures of mechanisms or robots are usually represented by its kinematic chain: each link of the mechanism is represented by a polygon whose vertices represent the joints (a binary link is represented by a line with two vertices, a ternary link is represented by a triangle with three vertices, and so on) and all the joints of the mechanism are assumed to be revolute or translational joints.

Fig. 1(a) shows a widely-used face-shovel hydraulic excavator, and Fig. 1(b) is the mechanism of its working device where link 1 is the base and link 7 is the end-effector. The kinematic chain corresponding to Fig. 1(b) is shown in Fig. 1(c).

The concept of topological graph in graph theory is frequently adopted to analyze the connectivity of kinematic chains in the literature. The topological graph can be derived from the kinematic chain by using vertices to denote the links of the chain and edges