Mechanism and Machine Theory 109 (2017) 231–249

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Approximate equations for the meshing stiffness and the load sharing ratio of spur gears including hertzian effects

Miryam B. Sánchez, Miguel Pleguezuelos, José I. Pedrero*

UNED, Departamento de Mecánica, Juan del Rosal 12, 28040 Madrid, Spain

ARTICLE INFO

Keywords:
Spur gears
Meshing stiffness
Load distribution
Load capacity

ABSTRACT

In this paper, the meshing stiffness of spur gear pairs, considering both global tooth deflections and local contact deflections, is evaluated at any point of the path of contact and approximated by an analytical, simple function. With this function, the load sharing ratio is calculated and compared with previous results obtained from the hypothesis of minimum elastic potential energy (MEPE model), considering the tooth deflections, but neglecting the hertzian deflections. Critical bending and contact stresses from both models are also compared both for standard and high contact ratio spur gears.

1. Introduction

The determination of the mesh stiffness and the load sharing of tooth gears is critical to predict the dynamic behavior or the load capacity of spur gear transmissions. Several models for the meshing stiffness and studies on the load distribution along the line of contact can be found in technical literature. Li [1] studied the effect of the tooth addendum on the contact and bending stresses of spur gears, and analyzed the tooth load and the load sharing rate, the transmission error and the mesh stiffness. Arafa and Megahed [2] evaluated the mesh compliance of spur gears by a finite element modeling technique, and discussed the load sharing among the mating gear teeth. Pimsarn and Kazerounian [3] developed a new method for estimating the equivalent mesh stiffness and the mesh load in gear system (pseudo–interference stiffness estimation).

More recently, Pedersen and Jorgensen [4] developed a method for the tooth stiffness estimation performed by using the finite element analysis. Fernández del Rincon et al. [5] presented a model for the meshing stiffness of spur gears taking into account global and local deformations, which was applied to study the influence of the profile shift on the efficiency [6]. Li [7] studied the influence of misalignments, tooth profile modifications and transmitted torque on the meshing stiffness and the load sharing ratio. Ye and Tsai [8] extended the study to high contact ratio spur gears. Marques et al. [9] computed the load sharing ratio by minimizing the elastic potential energy, considering frictional effects, and applied the obtained load distribution to compute the friction power losses. Marimuthu and Muthuveerappan studied the load sharing on spur gears with asymmetric profiles for standard [10] and high contact ratio [11] spur gear drives. Chen and Shao [12] calculated the mesh stiffness of spur gear pairs with profile modifications and tooth crack, and developed the dynamic simulation of the tooth with crack propagating [13]. Yu et al. studied the effect on the stiffness of the crack propagation [14] and the corner contact effects [15]. Cui et al. [16] studied the meshing stiffness and vibration response of cracked gears based on the universal equation of gear profile. Ma et al. [17] provided a method for the determination of the mesh stiffness of spur gears with tip relief.

A model of load distribution for external gears, based on the minimum elastic potential energy criterion, has been developed by the authors in previous works [18,19]. The elastic potential energy of a pair of teeth at any contact position of the path of contact was

* Corresponding author.
E-mail address: jpedrero@ind.uned.es (J.I. Pedrero).

http://dx.doi.org/10.1016/j.mechmachtheory.2016.11.014
Received 23 September 2016; Received in revised form 28 November 2016; Accepted 29 November 2016
Available online 07 December 2016

Nomenclature

b	Face width, mm
C_s	Shear potential correction factor
E	Modulus of elasticity, MPa
e	Tooth thickness, mm
F	Load, N
G	Transverse modulus of elasticity, MPa
h_a	Tooth addendum coefficient
h_F	Bending moment arm, mm
K_M	Meshing stiffness, N/mm^2
K_{TP}	Stiffness of the couple of teeth, N/mm^2
k	Stiffness, N/mm^2
m_n	Normal module, mm
R	Load sharing ratio
r_b	Base radius, mm
r_c	Contact point radius, mm
s_F	Tooth thickness at the critical section, mm
u	Gear ratio
x	Rack shift coefficient
Y_S	Tooth correction factor (ISO 6336–3)
y	Coordinate along the tooth centerline from the gear rotation center, mm
z	Number of teeth
Z_E	Elasticity factor (ISO 6336–2)
α_C, α_F	Load angle
α_n	Standard normal pressure angle
α'_t	Operating pressure angle
γ	Tooth angular thickness
ε_α	Transverse contact ratio
ρ	Relative curvature radius, mm
σ_F	Tooth-root bending stress, MPa
σ_H	Contact stress, MPa
ξ	Involute profile parameter

calculated by the integration of the equations of the theory of elasticity, as a function of the normal load. The load sharing among several spur tooth–pairs in simultaneous contact was obtained minimizing the total potential (which is the sum of the potential of each pair at its respective contact point and loaded with its respective load), regarding the restriction of the total load to be equal to the sum of the load at each pair. This model -named Minimum Elastic Potential Energy (MEPE) model- was applied to the calculation of the bending and pitting load capacity of standard [20–22] and high contact ratio [23,24] external gears.

One of the most interesting results of this investigation was an approximate, accurate equation for the inverse of the elastic potential for unit load and face width (named the inverse unitary potential) of the tooth pair, which was expressed as a function of the transverse contact ratio by means of a very simple formula [18,19]. This formula allowed expressing the meshing stiffness and the load sharing ratio by simple, analytic equations which made easier the studies of maximum stresses and load capacities. However, this model was based on the bending, shear and compressive elastic potential energy, but the hertzian contact deflections were not accounted. The influence of these contact deflections on the load sharing is small [5,9], because the contact stiffness is much greater than the combined bending, shear and compressive stiffness, and consequently the obtained values of the critical stresses [20–24] are valid for strength calculations.

Nevertheless, the contact stiffness is not complicated to be taken into account if assumed the Hertz's contact model between two cylinders is accurate enough to describe the contact between involute teeth. In fact, the hertzian deflection between two cylinders is proportional to the total tightening load, which means a load-independent stiffness.

Contact between involute spur gear surfaces occurs along a straight line, parallel to the gear rotation axes, and all the contact points of each surface have the same curvature radius. As these contact conditions are very similar to those corresponding to contact between two cylinders, the Hertz's model should be accurate. Li [1] obtained by FEM the load distribution along the width of the contact zone of two spur teeth and concluded that the Hertz formula is accurate for contact stress and contact width calculations of the gears. AGMA [25,26] and ISO [27] standards evaluate the contact stress with the Hertz equation for calculating the load capacity of involute spur and helical gears. Fernández del Rincon et al. [5] use the same formulation for the local deformations. Coy and Chao [29] use the Hertz solution to estimate the element size for a finite element study. Wang and Zhang [30] calculated the tooth mesh stiffness from the Hertz model of line contact between cylinders. In all the cases results were satisfactory, so it can be concluded that the Hertz equations for contact between two cylinders describe the contact between tooth surfaces accurately.

This paper presents an enhanced model for the spur gear mesh stiffness, including bending, shear, compressive and contact deflections. An adjusted equation for the inverse unitary potential (or for the mesh stiffness for unit face width) is also presented. As the previous one, the approximate equation is simple, accurate and depends on the contact ratio exclusively. With this equation the load sharing ratio, and accordingly the load at any point of the path of contact, has been determined, both for standard and high contact ratio spurs gears. Critical stresses and determinant load conditions are calculated, and discrepancies with previous values are discussed and eventually adjusted.

2. Load sharing model

The MEPE model [18,19] is based on the assumption that the couples of teeth in simultaneous contact are loaded in such a way that the elastic potential energy is minimum. This model, which is described in depth in [18], was developed considering the bending, shear and compressive deflections, but neglecting the contact deflections. In addition, results were expressed in terms of the unitary potential, which is defined as the elastic potential for unit load and unit face width.

Of course, the same results are obtained if the problem is approached in terms of stiffness instead of potential energy. In this section the MEPE model will be outlined in terms of stiffness, in order to facilitate the inclusion of the contact stiffness.