Mechanism and Machine Theory 109 (2017) 278–295

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Research paper

Multi-objective micro-geometry optimization of gear tooth supported by response surface methodology

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ARTICLE INFO

Keywords:
Gears
Optimization
Micro-geometry
Mechanical transmission
Response surface
Transmission error

ABSTRACT

This paper shows the application of response surface methodology for gear optimization using micro-geometry profile modifications. The suitability of three distinct metamodeling techniques is studied in this paper: Gaussian Process (stochastic), Shepard k-Nearest (nonparametric deterministic) and Polynomial (parametric deterministic). The described optimization strategy is implemented and tested on a case study consisting of a pair of identical spur gears, in which the goal is to find optimal micro-geometry modifications of tooth profile, providing decreased values of peak-to-peak transmission error and maximal contact stress along the meshing cycle, while maintaining the safety coefficient linked to tooth bending fatigue above a required threshold. The gear pair is analyzed under three different loading scenarios. It is shown that the described optimization strategy allows finding optimal micro-geometry modifications of tooth profile that enable significant improvements in all the observed performance indices with respect to the unmodified gear design, as confirmed by detailed numerical simulations of the optimal gears.

1. Introduction

With the development of numerical simulation methods, gear structural optimization has gained much attention by mechanical engineers, since computer – based analyses allow selecting an optimal design solution from a vast number of trial models, with limited need for costly and time consuming physical prototyping and testing. In order to improve gear operating parameters, tooth micro-geometry is often tuned to ameliorate the meshing process by eliminating flank-tip contact and, consequently, allowing loaded teeth for a smooth changeover. Currently, these issues are most frequently approached by using Finite Element Method (FEM) simulations, which can cover all the aspects of gear shape, position and operating conditions [1].

However, although the principles governing gears operation are well established and understood, detailed numerical analyses aimed at predicting their dynamic behavior and durability are still cumbersome and time consuming. This is because high fidelity approaches to gear numerical simulations require accurate description of local (i.e., contact stress) and global (i.e., tooth and body deflections) phenomena, which simultaneously influence the overall transmission behavior.

When it comes to optimization, it is commonplace to carry out these tasks by using analytical, semi-analytical or FE-based gear models. Artoni et al. [2] presented optimization methodology for cylindrical gears, based on micro-geometry modification of tooth, evaluating peak value of contact stress and peak-to-peak value of loaded transmission error (TE). Harianto and Houser [3] described a method for assessing the influence of gear geometry modification on various performance indices, including tooth root and contact

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http://dx.doi.org/10.1016/j.mechmachtheory.2016.11.015
Received 6 June 2016; Received in revised form 28 November 2016; Accepted 30 November 2016

stress and gear noise and vibration. Artoni et al. in Ref. [4], described tooth geometry optimization in hypoid gears based on generalized tooth flank description, using the so-called ease-off parameterized surface. Bonori et al. [5] presented an implementation of genetic algorithms for spur gear optimization, aiming at minimizing the generated vibrations. The goal of the procedure was to decrease the peak-to-peak value of static transmission error (STE) using tip and root reliefs. As shown by Parker et al. [6], STE, which is simpler to calculate, can be used to predict gear dynamic behavior. According to the findings described by Cai and Hayashi in Ref. [7], peak-to-peak STE (PP STE) is strongly correlated with gear dynamics. Therefore, minimization of the PP STE results in improvements of gear vibration behavior.

The optimal value of micro-geometry modifications is strongly correlated with the operational conditions, for which the transmission is designed. In other words, tip and root reliefs, which are optimal for a given loading torque, can perform worse than non-modified profile under a different load. Indeed, Faggioni et al. [8] used tooth profile modifications to perform a global optimization of gear vibrations in a wide range of operating conditions. A study on tooth profile modification was also discussed by Fernández et al. [9]. The authors took under consideration profile reliefs and manufacturing inaccuracies, showing their influence on the STE peak-to-peak values.

In case of simulation data that is difficult or long to predict and collect, the response surface methodology (RSM) can be usefully employed. This approach, which was studied extensively in the last decades, is based on the idea of substituting complex mathematical models by simpler, easy to compute representations that allow to predict the observed quantity values (i.e., the dependent variables) as a function of certain independent variables. In the result, a so-called surrogate or metamodel is generated, referred sometimes to as a 'model of a model'. A parameterized approximating function can be determined a priori (e.g., polynomial model) or not considered at all (e.g., Gaussian Processes or k-Nearest approaches), depending on the chosen algorithm. The latter alternative is perceived as more general and is often used when the output data exhibits a high degree of non-linearity or its form is simply unknown. The implementation of RSM to time consuming engineering optimization problems can be found in [10,11]. A description of various variants of surrogate-based data approximation was presented in [12,13].

Despite its great popularity in engineering, the RSM implementation to gear design, specifically to micro-geometry optimization, has gained scant popularity. Very limited information on its application is available in the literature dedicated to mechanical transmissions. One example can be found in Ref. [14] by Zhang et al., in which Kriging method combined with Latin Hypercube Sampling (LHS) design of experiment (DoE) was used successfully to predict strength responses of large scale gears. Surrogates were used in this work to perform global optimization with genetic algorithms (GA), finding an optimal set of macro-geometrical parameter values. The authors emphasized the significance of metamodeling to simplify engineering problems which, due to detailed numerical simulations, are becoming increasingly time-demanding nowadays. Kayabasi and Erzincanli in Ref. [15] used polynomial-based RSM to perform shape optimization of a harmonic drive. Other examples of using RSM can be found in [16,17], in which data approximation techniques support optimization of gear manufacturing, aimed at improving the process of tooth profile generation. Zhang and Guo in [16] used second order polynomial regression to predict dynamic transmission error fluctuations calculated using an analytical model of planetary gearbox. Park in [17] used the same type of polynomial approach to predict variation of a static transmission error computed for helical gears.

Limited number of literature positions on the subject of RSM-based gear optimization can be identified as an open research gap and allows to pose a question on usefulness of metamodeling in this class of structural optimization problems. This paper aims at providing an insight into this issue and demonstrates the applicability of metamodel-based multi-objective gear optimization on a real-life case study. The RSM-based optimization was carried out on a pair of identical spur gears to obtain improvements in PP STE and contact stress, with a constraint on the fatigue safety coefficient for tooth bending. Each gear design was altered by micro-geometry modification and assessed by non-linear static FEM simulations. Three different metamodeling techniques were used to build the surrogates for every output quantity, namely: Gaussian Processes (GP), 3rd order polynomial model (PL) and Shepard k-Nearest (SKN). Each of the response surfaces was validated and the selected, most accurate metamodels were used in conjunction with multi-objective genetic algorithm (GA) in order to find a set of parameter values for optimal microgeometry modification. To make the optimization attempt more comprehensive, the objectives were settled for three different loading torque values: 350 Nm, 500 Nm and 650 Nm.

The paper is structured as follows: Section 2 explains the idea of multi-objective optimization and discusses the use of population-based metaheuristics for solving this type of problems. Selected metamodeling techniques and response surface validation indices are introduced to the reader in Section 3. Section 4 presents the RSM-based optimization process workflow. The concept of micro-geometry modifications is described in Section 5. Section 6 describes the case study numerical model used for FEM simulations and discusses the observed output quantities. Section 7 presents the generated metamodels and provides details on assessment of the accuracy for different data approximation methods. Section 8 presents the optimization assumptions and describes the relationships between the goals of the procedure. Section 9 presents the obtained surrogate-based quasi-optimal solutions and their numerical validation. Section 10 concludes the paper.

2. Multi-objective optimization

Following Coello et al. [18], a multi-objective optimization problems (MOP) can be defined by Eq. (1):

$$\textit{Objective}:\quad \min/\max[f(\mathbf{x})],\quad \mathbf{x}\in\Omega \textit{Subject to}:\quad h_i(\mathbf{x})=0,\quad g_j(\mathbf{x})\le 0,\ i=1,\dots,l,\quad j=1,\dots,t \tag{1}$$

where $f(\mathbf{x})=[f_1(\mathbf{x}),\dots,f_k(\mathbf{x})]$ is a vector of k objective functions, subject to l equality $h_i(\mathbf{x})=0,\ i=1,\dots,l$ and t inequality