Mechanism and Machine Theory 110 (2017) 16–26

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Function generation with two loop mechanisms using decomposition and correction method

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ARTICLE INFO

Keywords:
Function generation
decomposition and correction method
Watt II linkage

ABSTRACT

Method of decomposition has been successfully applied to function generation with multi-loop mechanisms. For a two-loop mechanism, a function y = f(x) can be decomposed into two as w = g(x) and y = h(w) = h(g(x)) = f(x). This study makes use of the method of decomposition for two-loop mechanisms, where the errors from each loop are forced to match each other. In the first loop, which includes the input of the mechanism, the decomposed function (g) is generated and the resulting structural error is determined. Then, for the second loop, the desired output of the function (f) is considered as an input and the structural error of the decomposed function (g) is determined. By matching the obtained structural errors, the final error in the output of the mechanism is reduced. Three different correction methods are proposed. The first method has three precision points per loop, while the second method has four. In the third method, the extrema of the errors from both loops are matched. The methods are applied to a Watt II type planar six-bar linkage for demonstration. Several numerical examples are worked out and the results are compared with the results in the literature.

1. Introduction

There are several methods for the kinematic synthesis of function generating mechanisms. Polynomial approximation methods such as interpolation, least squares and Chebyshev approximation methods are some of the commonly used methods [1]. Among these, interpolation approximation method is the easiest one to implement. On the other hand, least squares approximation method provides more accurate approximation with respect to least squares (or L2) norm and Chebyshev approximation method provides more accurate approximation with respect to Chebyshev (or $L\infty$) norm, provided that these two methods are applied for a single loop function generator mechanism.

The approximation accuracy in function generating mechanism synthesis can be enhanced by increasing the number of design parameters of the mechanism. This can be accommodated by either of the following methods: 1) introducing artificial frames of reference for the input or output of the mechanism [1]; 2) also considering the amount of displacements of the input or the output link [2]; 3) combining linkages with gears [3]; 4) using a mechanism with more design parameters [4]; 5) using additional loops for a selected mechanism [5]. There are several studies on function generating planar six-link mechanisms. Svoboda [6] considered a Watt II type six-bar linkage as a double three-bar linkage and specified nine design parameters (three link lengths per loop–taking scaling into account and three rotation ranges for the three links connected to the ground). He formulized the function generation problem as composition of two functions corresponding to the two loops, which we call the method of decomposition. He proposes alternative uses of the approach. The first use is such that the first loop is designed to approximately generate the desired function

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http://dx.doi.org/10.1016/j.mechmachtheory.2016.12.004
Received 19 February 2016; Received in revised form 14 December 2016; Accepted 17 December 2016

and the second loop is used to tune the result. In the second use, the desired function is decomposed into two identical functions such that the square root of the function is generated by each of the identical four-bar loops with equal input and output travels. Svoboda calls this latter method as the method of successive approximations. The generalized decomposition method proposed by Alizade et al. [7] allows the decomposition of the function arbitrarily. This enables an extra design parameter for the designer. McLarnan [8] utilized an iterative numerical method for synthesis of planar six-bar linkages for at most 9 precision points with Watt linkages and at most 11 precision points with Stephenson linkages. Rao et al. [9] utilized Burmester theory in order to design six-bar linkages performing function and path generation simultaneously. Several examples of such dual-purpose (combination of function, path, motion generation with the same mechanism) are presented in [10]. Dhingra and Mani [11] derived the input/output (I/O) relationship for Stephenson III and Watt II type planar six-bar linkages and used Newton-Raphson numerical method to solve the function generation synthesis problem with 9 and 11 precision points. Dhingra et al. [12] used homotopy methods for function generation with planar six-bar linkages.

Liu et al. [13] made use of homotopy methods for function generation with six-bar linkages for five precision points. Simionescu and Alexandru [14] worked on the optimal design of Stephenson linkages by increasing the degree-of-freedom to two by removing one of the links. [15] devised a modular approach for design of six-bar function generators. Shiakolas et al. [16] devised a methodology that combines differential evolution and geometric centroid of precision positions technique in order to perform synthesis of Stephenson III type six-bar linkages for dwell and dual-dwell mechanisms with prescribed timing and transmission angle constraints. Kinzel et al. [17] used the so-called geometric constraint programming (GCP) to design a Stephenson III type planar six-bar linkage for function generation with up to 11 precision points. Both graphical and analytical methods are used in GCP and it makes use of commercially available CAD packages to simultaneously meet precision point conditions. Hwang and Chen [18] applied constrained optimization techniques for designing Stephenson II type function generators avoiding order, circuit, and branch defects. Sancibrian [19] made use of an improved version of the generalized reduced gradient optimization method for function generation synthesis of several planar linkages including the Stephenson II, Stephenson III and Watt II type six-bar linkages.

Plecnik and McCarthy [20] also worked on Stephenson II type type of six-bar linkages for function generation with eight precision points. By assuming some of the link lengths, a set of 22 equations with a total degree of 705,432 is obtained. Later on, Plecnik and McCarthy [21] also worked on function generation with a Stephenson II type planar six-bar linkage for 11 precision points. The loop closure equations constitute a set of 70 quadratic equations and the system is reduced to 10 eighth-degree polynomials. The resulting set of equations have a total degree of 1.07×10^9. In both of the last two studies, the equations are solved using continuation method. The latter study resulted in 1,521,037 nonsingular solutions. Agarwal et al. [22] used a genetic-algorithm-based multi-objective optimizer for function generation with a Stephenson-III type planar six-bar linkage. In addition to the structural error defined based on the I/O relationship, the derivative of the structural error is also taken into account, therefore the formulation is called the dual-order formulation. Also, analytical conditions are derived for the identification of the candidate designs which are free of singularities, mobility or branch defects. The numerical examples result in comparable values with the ones that are presented in [20].

All methods mentioned above are based on numerical methods, whereas an analytical formulation is presented in this study. The drawback of the proposed method is that relatively few number of precision points are used. The powerful side of the analytical formulation is that the designer can carry out hundreds of trials in several minutes. Furthermore, the methods proposed in this study allow the designer to tune the design while monitoring several properties such as link length ratios, transmission angle and etc.

The decomposition method is applicable when there are multiple loops in the generator mechanism and is based on decomposition of the function to be generated into as many functions as the number of loops. Maaroof and Dede [23,24] have worked on application of the interpolation approximation for a Bennett 6R linkage using the decomposition method. Although interpolation approximation seems to result in inferior results compared to the other two approximation methods, Maaroof et al. [25] showed that superiority of one method over the others is lost when the decomposition method is used.

In this study we apply the decomposition method to a Watt II type planar six-bar mechanism. We propose three correction methods to improve the accuracy of function generation. These methods can be easily and conveniently applied to other two-loop mechanisms, as well. The proposed methods are based on correction of the function generation errors of the first loop in the second loop. Interpolation approximation is used as the synthesis method.

2. Description of the mechanism and function synthesis problem

The Watt II type planar six-bar mechanism is composed of two ternary and four binary links connected to each other by seven revolute joints. In Fig. 1, the input of the mechanism is the ϕ angle and the output is ψ angle. γ angle is the output of the first loop and will be used as an intermediate variable. A Cartesian coordinate frame with origin at A_0 is to be used such that the x-axis passes through B_0. The input angle ϕ and the intermediate angle γ are measured from their respective fixed reference axes which make angles of ϕ^* and γ^*, respectively with the x-axis. The fixed reference axis of the output angle ψ makes an angle of ψ^* with respect to B_0D_0 direction. In general, ϕ^*, γ^* and ψ^* are design parameters to be determined via synthesis. Since the I/O relationship of a mechanism with revolute joints does not change when the mechanism is scaled, the four bar loops A_0ABB_0 and B_0CDD_0 can be independently resized arbitrarily. So, without loss of generality we may assume $|A_0B_0| = |B_0D_0| = 1$ for the fixed link lengths. The other link lengths of the mechanism are denoted as $|A_0A| = a$, $|AB| = b$, $|B_0B| = c$, $\parallel BB_0C = \alpha$, $\parallel D_0B_0x = \beta$, $|B_0C| = d$, $|CD| = e$ and $|D_0D| = f$. A careful inspection shows that angles α and β are not independent design parameters, but $\alpha+\beta$ is effective in the I/O relationship of four-bar loop B_0CDD_0, because the effective input of the loop is $\parallel CB_0D_0 = \gamma + \gamma^* - (\alpha + \beta)$. β also contributes to the