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Synthesis of three-revolute spatial chains for body guidance

Jonathan D. Hauenstein[a,*], Charles W. Wampler[b], Martin Pfurner[c]

[a] University of Notre Dame, Notre Dame, IN, USA
[b] General Motors R & D, Warren, MI, USA
[c] University of Innsbruck, Innsbruck, Austria

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ABSTRACT

For a given mechanism type, the solution set of a body guidance synthesis problem comprises all mechanisms whose end-effector can reach a set of prescribed poses (position and orientation). For three-revolute spatial chains, five general poses will yield a synthesis problem having only finitely many solutions, while specifying fewer than five poses leads to higher-dimensional solution sets. We use numerical algebraic geometry to compute solution sets for two to five general poses, and in particular, we find, for the first time, that the five-pose synthesis problem generically has 456 solutions. We also show how our results agree with and extend results in the literature.

1. Introduction

The problem of synthesizing mechanisms that guide their end-effector through a number of prescribed discrete poses (position and orientation) has a long tradition in kinematics with the some of the first formulations and solutions by Schoenflies [1] and Burmester [2]. Beginning in the second half of the 20th century, various planar and spatial synthesis problems were formulated, e.g., see [3], with McCarthy [4] providing a good general overview.

Whereas a six degree-of-freedom robot, such as a serial-link six-revolute (6R) arm or a Stewart-Gough parallel-link robot (i.e., type 6(SPS)),[1] can carry a payload through an arbitrary number of poses within its reachable workspace, mechanisms with fewer degrees of freedom also have many uses. Since a lower-degree-of-freedom device cannot reach arbitrary poses, one approach to designing such mechanisms is to specify a finite number of poses that must be reached and then solving the associated kinematics equations to find mechanisms that can reach those poses.

In this paper, we consider the synthesis of serial three-revolute (3R) spatial chains. A simple dimension count indicates that five is the maximum number of general end-effector poses for which a solution will exist. Thus, we consider the synthesis problems arising from 2, 3, 4, and 5 prescribed general poses and solve them using numerical algebraic geometry. For 5 general poses, the synthesis problem has a finite number of solutions whereas fewer poses yield an infinite number, i.e., a positive-dimensional solution set.

While a 3R synthesis problem might arise out of the desire to use a 3R robot for a task, a method of solving such problems has many other potential uses. As an example, consider guiding body B with respect to body A using a one degree-of-freedom mechanism through 5 poses. One option is to use a spatial 3-loop mechanism that connects B to A via one 3R chain and two S-S pairs. It has been shown, e.g., see [5,6], that can construct S-S pairs for motion through up to 7 poses. Combining two such S-S pairs with a solution to the 3R synthesis problem gives the complete synthesis of the mechanism. One could consider many other arrangements that include a 3R chain.

* Corresponding author.
[1] Typical joint types: “R” for revolute, “P” for prismatic, and “S” for spherical.

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Previous work on the 3R synthesis problem has been published by Lee and Mavroidis [7–9], also with Merlet in [10]. For the 3-pose and 4-pose cases, they restrict to problems having a finite solution set by fixing certain design parameters: six parameters for the 3-pose case and three parameters for the 4-pose case. The resulting synthesis equations were solved using either homotopy continuation [7,8] or elimination methods [9]. Whereas those efforts used the polynomial nature of the equations to compute complete solution sets using complex numbers, the 5-pose case was treated in [10] by using interval analysis to find all solutions in a specified interval box for one particular 5-pose synthesis problem.

These prior publications all formulate the synthesis problems as a system of polynomial equations arising from a matrix equation using the Denavit and Hartenberg [11] convention of modeling spatial linkages. We develop an alternative formulation, more amenable to numerical solution, based on a direct determination of the location of the joint axes. This allows us to solve the 3- and 4-pose problems in a more general way, without resorting to pre-specifying mechanism parameters, and to completely solve the 5-pose problem in the complex domain. We describe the relationship between the two formulations and replicate the earlier results with this new formulation.

Our general solutions in the complex domain allow us to efficiently carry out subsequent calculations by the methods of parameter continuation and regeneration, which we also describe. In our computations, we use the software package `Bertini` [12]. For more information about using homotopy continuation and numerical algebraic geometry applied to kinematics, see Wampler and Sommese [13] and Sommese, Verschelde and Wampler [14].

The remainder of the paper is as follows. Section 2 presents our reformulation of 3R synthesis as a system of polynomial equations. We briefly review numerical algebraic geometry in Section 3 and describe its use in Section 4, to completely solve the synthesis problems of 3R spatial chains for 2, 3, 4, and 5 general poses. We apply the methods to solve numerical examples in Section 5. A discussion about the results and conclusion of this paper are presented in Section 6.

2. Problem formulation

We begin by reviewing the Denavit-Hartenberg (D-H) conventions for modeling a serial-link chain. The works of Lee and Mavroidis on 3R synthesis were formulated in terms of a product of transformation matrices populated by D-H parameters, including joint angles. We start with the same underlying link geometry, but eschew joint angles and instead use joint vectors that directly indicate the linkage conformation at each specified pose.

Denavit-Hartenberg parameters describe the relative positions of two arbitrary lines in space, see Fig. 1, which will be the axes of adjacent revolute joints in a serial chain. Two coordinate frames Σ and Σ' are attached to the two lines such that the z-axis of Σ is aligned with the first line and the x-axis is parallel to their common normal. Coordinate frame Σ' has its origin at the footpoint of the common normal on the second line, with its z-axis aligned with this line and its x-axis aligned with the x-axis of Σ. Parameter a denotes the distance between the lines, d is the offset of the common normal on the first line to the origin of Σ and α is the twist angle. The relative transformation from Σ to Σ' can be written as consecutive transformations of the form: translation along the z-axis with distance d, followed by a translation along x-axis with distance a, and then a rotation around x-axis by an angle α.

When the D-H convention is used for the description of a serial chain with n revolute joints, one is able to write the pose of the end effector frame Σ_{n+1} with respect to the base frame Σ_0 in terms of relative transformations from Σ_i to Σ'_i together with, for each $i = 1,\ldots,n$, a transformation from Σ'_{i-1} to Σ_i that is a rotation around z-axis i by joint angle θ_i. For a fully general chain, one must also include angle θ_0 as a mechanism parameter associated to the placement of the mechanism in the world frame and parameters θ_{n+1} and d_{n+1} associated to the placement of the end-effector frame in the final link. Altogether, there are $3n + 6$ mechanism parameters, namely

$$\theta_0,\ \{(d_i,\ a_i,\ \alpha_i),\ i = 0,\ldots,n\},\ \theta_{n+1},\ d_{n+1}$$

and there are n joint angles, θ_i, $i = 1,\ldots,n$. For N general poses and an n-revolute chain, the synthesis problem has $3n + 6 + Nn$ unknowns and $6N$ constraints. In particular, these are equal when n=3 and N=5. For comparison to the work of Lee and Mavroidis on 3R problems, we note that they rename the last two parameters as $\phi = \theta_4$ and $d = d_4$.

From this point forward, we focus our discussion on 3R spatial chains, i.e., n=3, with N given poses. For $i = 1,\ldots,N$, let pose i be

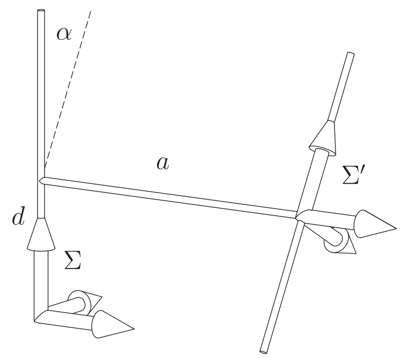

Fig. 1. Denavit-Hartenberg parameters.

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