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Modeling and optimal design of circular-arch flexible structure with radial-freedom considering geometry and material selection simultaneously

Yuyan CAO*, Zhichen WANG, Chao ZHOU

Department of opto-electrical detection, Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Jilin Province 130033, China

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ABSTRACT

The circular-arch flexible structure is widely used in various fields, especially the support structure of the optical mirrors. This paper aims to present a generalized formulation of the circular-arch flexible structure and a continuous method to optimize this flexible structure considering the material selection and geometry simultaneously. First, an analytical model based on the variational principle is derived for calculating the radial and tangential stiffness of the flexible element, and then the generalized formulation of the integral flexible structure is obtained by considering force equilibrium and compatible deformation. Second, the structural optimization is implemented by combining the material selection and geometrical parameters, where the continuous artificial variables are used to represent the selected material. Finally, the experimental and numerical examples are given to verify the analytical formulation and the optimization scheme. The experimental and FE simulation results of the flexible element and the integral flexible structure indicate that the presented mechanical model is capable of capturing the linear behavior. For the geometrical nonlinear deformation, there exist some errors. And the optimization results demonstrate that the presented scheme is able to obtain the discrete material design and the optimal geometry.

1. Introduction

The circular-arch flexible structure presented in this paper is composed by several identical shallow arches that act as the flexible elements. It can be seen as a special type of flexure hinge with radial freedom. Flexible structures have been widely used in a large number of fields with high precision requirements, such as optical structure [1], gyroscopes [2], actuators and sensors [3], micro/nano precision positioning stages [4,5], grippers and manipulators [6]. The wide-spread use of these precision flexible structures is due to the advantages they offer: (i)achieving high precision and repeatability since there is no friction, (ii)being easy to fabricate and maintain, which results in its low production cost [7].

Despite these advantages mentioned above, the mechanical model of the flexible structure is very complicated especially when many of them work together [8]. Many mathematical models have been developed to describe the mechanical behavior of the flexible structure. Paros and Weisbord [2] introduced an analytical model for calculating the compliances of single-axis and two-axis circular flexure hinges with constant cross-section in terms of deflections and rotations produced through bending and axial loading in their fundamental work. According to the theory of Paros and Weisbord [2], Smith et al. [9] introduced the approximate compliance equations for symmetric elliptic flexure hinge. Wu and Zhou [10] adopted the integration of linear differential equations of a beam to derive the concise compliance equation for flexure hinge. Lobontiu et al. [11] developed closed-form compliance equations for symmetric corner-filleted flexure hinges by using Castigliano's second theorem and made a comparison with the right circular flexure hinges, the results of which were confirmed by the finite element simulation and experiments. Lobontiu and Garcia [12] proposed a new type of two-axis flexible structure and developed a generic formulation in terms of the geometric curves defining the two notches. Lobontiu and Garcia [13] have formulated a closed-form equation for displacement and stiffness calculation of the planar compliant mechanisms using the strain energy and Castigliano's displacement theorem, and studied the performance of amplifying compliant mechanisms based on Lagrange's multipliers and Kuhn-Tucker conditions. Hopkins and Culpepper [14] [15] proposed freedom

* Corresponding author.

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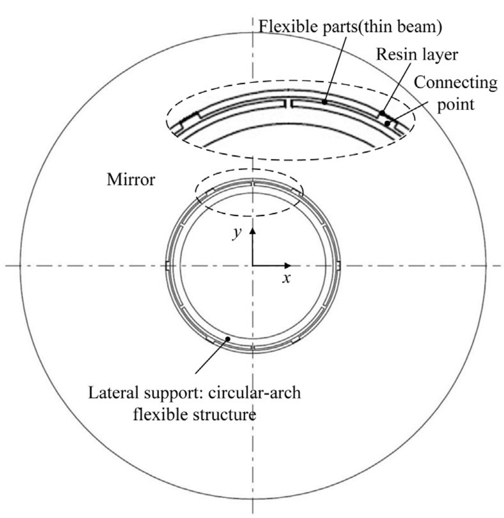
and constraint topologies(FACT) synthesis method for compliant mechanism design, which utilizes a comprehensive library of geometric shapes that allow designers to visualize the quantitative relationships between all possible compliant mechanism design concepts and all possible motions for any given design problem. Hao [16] designed a class of single-axis translational flexure guiding mechanism. The wire beams were used as the distributed compliance module, which could avoid constraint stiffness decrease over the primary motion. However, most of them mentioned above mainly focus on the flexible structures with various shapes, and the researchers prefer to use the finite element method to verify the analytical model due to the limited experimental data available [12,17,18]. The circular-arch flexible structure, which is widely used in optical equipments, has received less attention. The concept of circular-arch flexible structure was first introduced by Yolder [19] and was adopted and improved by several researchers [20–22]. Yolder takes advantage of the radial freedom of such flexible structure to design the lens mounting structures. This flexible structure can be seen as the combination of discrete flexible elements, either the straight beam or the curved one. However, few researches focus on its analytical theory. Lim et al. [23], using Euler-Bernoulli theory and Timoshenko theory, derived the bending solution of a curved beam which can be seen as an element of this circular-arch flexible structure. Ahuett-Garza et al. [24] explored the use of curved beams as large displacement hinges in planar compliant mechanisms to overcome the limited deformation range of conventional hinge, and introduced the analytical expressions that predict deflections under different types of loads.

Designing this circular-arch flexible structure includes selecting the best materials and determining the optimal geometrical parameters. In order to obtain the optimal combination of geometry and material, the geometry optimization and material selection should be considered simultaneously. As mentioned in the literatures [25] [26], the material selection problem refers to the discrete optimization problem that is not amenable to the gradient-based algorithms, thus its solving efficiency is low. To avoid this, the discrete material selection should be relaxed to the continuous optimization problem. Hvejsel and Lund [25] and Hvejsel et al. [26] presented two multi-material interpolation schemes to relax the original discrete material selection problem, and then realized the simultaneous topology and material design. Kennedy and Martins [27] presented a relaxation technique for the layered composite structures with discrete layer ply-angles. The linear and nonlinear constraints are introduced to force the continuous artificial variables to attain the values 0/1. Kennedy [28] proposed an efficient gradient-based algorithm for the discrete thickness optimization problems. The piecewise constraint penal functions are imposed on the intermediate designs. Stegmann and Lund [29] proposed the discrete material optimization methods for multi-material distribution and lamination sequence design problems.

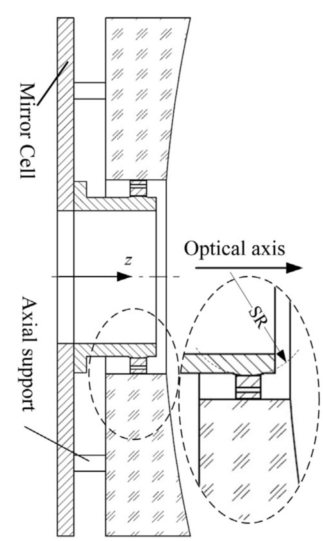
This paper is organized as follows. In Section 2, the mechanical model of the flexible element is derived using the variational principle. Based on the element model, the generalized formulation of the integral flexible structure is obtained. The implementation of the flexible structure optimization considering the material selection and geometry simultaneously is presented in Section 3. The experimental and numerical verification examples are described in Section 4. Finally, the conclusions are discussed in Section 5.

2. Mechanical formulation

It is known that the mirror's surface precision exerts great effect on the performance of the optical system and is determined by many factors, where the support structure is a key one [30]. The mirror, especially for the large one, needs a complicated support structure to hold it in the required surface precision and in the proper position regardless of the thermal or mechanical stress of the structure [31]. This indicates that the presence of mechanical constraints limits the motion of the mirror. The ideal mechanical constraints, including the lateral and axial constraints, should be kinematic, as Fig. 1 shows. That is to say all six degrees of freedom(DOFs) (three translations and three rotations, *dx*, *dy*, *dz*, *Rotx*, *Roty*, *Rotz*) would be independently constrained without any redundancy [19]. In order to fully constrain the mirror's motion, the support structure should introduce six constraints. From Fig. 1, it can be found that the axial support constrains three DOFs, *dz*, *Rotx*, *Roty*, and the lateral support constrains two DOFs, *dx*, *dy*. The last DOFs, *Rotz*, is constrained by the rotational constraint, which is not shown in Fig. 1. Since it contacts with the other structural component through the spherical surface(see Fig. 1b), the lateral support would not introduce other constraints except *dx* and *dy*. Usually, the mirror and its support structure have different thermal expansion coefficients(CTEs), so the thermal stress between their contact surface will be inevitable. Since the stiffness of the circular-arch



(a) top view



(b) lateral view

Fig. 1. The mirror and its supports

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