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Output feedback integral control of piezoelectric actuators considering hysteresis

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ABSTRACT

In this paper, output feedback integral control of piezoelectric actuators is considered with respect to the hysteresis effect. The linear dynamics of the piezoelectric actuator is modeled as a linear state space system with an input nonlinearity that considers the hysteresis effect. A proof of the Lyapunov stability of the system with integral control is presented, and a method for deriving the upper bound for the regulating gain is shown. A simple example is used to illustrate the approach, and then the approach is applied for tracking a step signal with an experimental single-axis piezoelectric actuator to verify that the system is stable.

1. Introduction

Many high-precision positioning mechanisms are now being based on piezoelectric actuators, in particular of stack type, due to their high resolution. This is due to the fact that the actuator can be integrated into flexure mechanisms, making the positioner monolithic in nature, and hence the stage does not experience either stiction or friction effects. The limitation on the range of movement for these actuators, which is on the order of micrometers, makes them natural for applications in a variety of nano/micro-positioning applications, of which atomic force microscopy is perhaps the most well known.

The actuator itself is driven through the converse piezoelectric effect, as an applied electric field generates mechanical strain in the material. However, this effect also has two nonlinearities that affect the positioning accuracy of the piezoelectric as an actuator. The first nonlinearity is known as creep, which creates a slow drift in the position of the actuator over longer time periods. This effect is most likely due to some dipoles in the material that are not currently aligned with the electric field, slowly becoming aligned with the applied electric field in time. There are several different types of models for creep of differing complexity [1], such as the fractional order model [2], and the logarithmic model [3]. However, as this effect is slower than the dynamics of the piezo, it is typically easy to mitigate with integral control. As the mechanical bandwidth of the actuators typically extends into the hundreds or thousands of Hertz, they can generate fast motions, however this precision is limited by the noise level in the closed loop system which includes both the electronics and the feedback control system used to mitigate the creep, and the effect of the second nonlinearity, hysteresis. This nonlinearity depends on the current and previous inputs to the actuator and is typically assumed to be rate-independent, which means that its behavior is independent of input frequency. Most commonly, phenomenological models are employed to model the hysteresis and the most common of these, among many types, are the Preisach, Prandtl–Ishlinskii, Maxwell slip, and Bouc–Wen models [4,5].

Possibly the most common approach for the control of piezoelectric stack actuators is to develop models of the hysteresis and creep effect, and then invert them for feedforward control [6,7]. This approach typically requires the precise identification of the full set of parameters that make up the model of choice. Of course, feedforward methods have their well-known disadvantages that they are not robust to disturbances or changes in the system. Feedback control on the other hand can be made robust to disturbances or changes in the system, however, typically at the expense of reducing the bandwidth of the system. A number of feedback controllers have been proposed including sliding mode [8,9], model reference adaptive control [10], a backstepping controller [11], and

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an integral plus vibration controllers [12]. Of course, feedforward and feedback control can also be combined [13] where ZVD input shapers were combined with PI control. Another method is where the dynamics of the actuator were linearized using high-gain feedback control [14] and then an inverse model of the linear vibrational dynamics was used to formulate a feedforward command. Another related approach is to invert the hysteresis inside the loop and then to design a feedback controller for the linearized dynamics, with a feedforward controller in front of the loop to extend the system bandwidth [15].

A problem in the analysis of feedback control for piezoelectric actuators is how to account for the hysteresis effect in the stability analysis of the system. Hence, piezoelectrics are typically assumed only to be represented by linear models, such as a low-order transfer function model that neglects consideration of the hysteresis. The hysteresis [16] was considered by employing an approximate inverse of it inside of the loop, and then the servo and stabilizing controller were designed to be robust to the uncertainties in the loop resulting from incomplete cancelation. Compounding the problem of considering hysteresis is the fact that these actuators all typically require some type of integral control action in order to achieve their desired position, as a constant voltage must be maintained for this to occur, and to counteract the creep effect. The problem with integral control is that the analysis is complicated by the presence of a simple pole at the origin, and hence, represents the critical case for stability. A state feedback based method of designing linear controllers that include an integrator state and that consider hysteresis was presented for magnetic shape memory alloys [17,18]. An analysis similar to that of the circle criterion [19] was developed in continuous time for an autonomous nonlinearity with a possibly time-varying integral gain, for a class of nonlinearities, which built off of the analysis of the discrete time case [20]. The objective of this paper is to simplify the analysis, to show that the hysteresis operator belongs to the class of nonlinearities considered, and then to apply the analysis to derive the maximal integral gain for output feedback integral control of an experimental piezoelectric actuator. The simplification serves to make the proof more useable in practice for the design of linear time invariant controllers with an integral term as it can explicitly consider the limitations imposed inside the control system loop by the hysteresis.

This paper is organized as follows. In Section 2, the dynamic model of a piezoelectric actuator is built considering the input nonlinearity. Section 3 presents the controller design and its stability proof. Experimental studies are included in Section 4 and Section 5 concludes this paper.

2. Dynamic modeling

The dynamic model of a piezoelectric actuator can be described by a linear transfer function under small signal conditions. For larger signals, a phenomenological model based on the modified Prandtl–Ishlinskii model of hysteresis can be used. Here, the piezoelectric actuators dynamic model can be represented as a combination of these two systems, as a single-input–single-output linear state space system, with the hysteresis considered as a nonlinearity on the input. This can be written as

$$\dot{x} = Ax + B\phi(u) \tag{1}$$

$$y = Cx + D\phi(u) \tag{2}$$

where $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, $C \in \mathbb{R}^{1\times n}$, $D \in \mathbb{R}$, and $u, y \in \mathbb{R}$, $\phi : \mathbb{R} \to \mathbb{R}$. With $G(s) = C(sI - A)^{-1}B + D$, this can be represented graphically as in Fig. 1.

The input nonlinearity of the piezoelectric actuator can be represented by the Prandtl–Ishlinskii phenomenological model of hysteresis. The basic block of the Prandtl–Ishlinskii operator can be given as

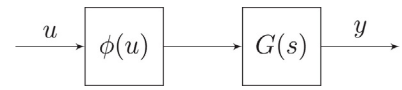

Fig. 1. Plant block diagram.

$$\psi_a(u, w) = \max\{u(t) - a, \min\{u(t) + a, w\}\} \tag{3}$$

where $u(t)$ is the control input, and a is the control input threshold value (the magnitude of backlash). The hysteresis operator can be defined recursively as [21]

$$\Psi_a[u](t) = \begin{cases} \psi_a(u(0), \epsilon) \\ \psi_a(u(t), \Psi_a[u](t_i)) & \text{for} \quad t_i < t \le t_{i+1} \end{cases} \tag{4}$$

where $0 = t_0 < t_1 < \ldots < t_{N-1}$ is a partition of $\mathbb{R}_+$ such that the function u is monotone on each of the subintervals $[t_i, t_{i+1}]$, and ϵ is the initial consistency condition which represents the internal state of the piezoelectric actuator before $u(0)$ is applied at $t = 0$. It is usually, but not necessarily, initialized to zero, representing that the actuator starts from a deenergized state. Here the hysteresis operator is defined as $\Psi_a[u]$, where it can possess an initial state. It is more commonly written as in [21], with a non-zero initial condition as $\Psi_a[u, \epsilon]$ to denote its dependence on its initial state.

The generalized operator is given by the weighted summation of a finite number of hysteresis operators as

$$\psi(u(t)) = w_h^T \Psi[u](t) \tag{5}$$

where $w_h^T = [\, w_{h_0} \quad w_{h_1} \quad \ldots \quad w_{h_m} \,]$ denotes the slope (or gain) of each individual backlash operator and $\Psi = [\, \Psi_{a_0} \quad \Psi_{a_1} \quad \ldots \quad \Psi_{a_m} \,]^T$ is the vector containing the individual backlash operators. Each of the backlash operators will have a threshold width of $2a$ beyond the initial loading curve with $0 = a_0 < a_1 < \ldots < a_m$. As $a_0 = 0$ it can be seen that the first operator will always be used as it describes the general linear response of the actuator weighted by a factor w_{h_0}. The subsequent operators are only used when the control inputs are greater than their respective threshold values a_i.

The standard Prandtl–Ishlinskii operator is symmetric about the center point of its loop, but in experiment it can be observed that the loop is in fact asymmetric. A saturation operator can be combined in series with the hysteresis operator to yield this asymmetric behavior [22]. The saturation operator can be taken as a weighted linear superposition of linear-stop or one-sided dead zone operators. The dead-zone operator is a nonconvex, asymmetrical, memory-free nonlinear operator [22] that can be given by

$$\Phi_d[\psi](t) = \begin{cases} \max\{\psi(t) - d, 0\} & \text{if} \quad d > 0 \\ \psi(t) & \text{if} \quad d = 0 \end{cases} \tag{6}$$

and the full response of the hysteresis becomes

$$\phi(u(t)) = w_s^T \Phi[\psi](t) = w_s^T \Phi[w_h^T \Psi[u]](t) \tag{7}$$

where $w_s^T = [\, w_{s_0} \quad w_{s_1} \quad \ldots \quad w_{s_p} \,]$ is the weight vector, $d = [\, d_0 \quad d_1 \quad \ldots \quad d_p \,]$ where $0 = d_0 < a_n < d_1 <. .. < d_p$ is the saturation threshold and $\Phi = [\, \Phi_{d_0} \quad \Phi_{d_1} \quad \ldots \quad \Phi_{d_p} \,]^T$. The threshold values a_i are usually chosen to be evenly spaced across the input range, however, the threshold values d_i of the saturation operator need not be equally spaced and can be difficult to obtain.

3. Integral control with input nonlinearities

The state space representation of a single-input–single-output system with input nonlinearities is given in Eqs. (1) and (2)