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# Synthesis of multiple degrees-of-freedom spatial-motion compliant parallel mechanisms with desired stiffness and dynamics characteristics

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ABSTRACT

This paper presents a new design method to synthesize multiple degrees-of-freedom (DOF) spatial-motion compliant parallel mechanisms (CPMs). Termed as the beam-based structural optimization approach, a novel curved-and-twisted (C-T) beam configuration is used as the basic design module to optimize the design parameters of the CPMs so as to achieve the targeted stiffness and dynamic characteristics. To derive well-defined fitness (objective) functions for the optimization algorithm, a new analytical approach is introduced to normalize the differences in the units, e.g., N/m or N m/rad, etc., for every component within the stiffness matrix. To evaluate the effectiveness of this design method, it was used to synthesize a 3-DOF spatial-motion ($\theta_x-\theta_y-Z$) CPM that delivers an optimized stiffness characteristics with a desired natural frequency of 100 Hz. A working prototype was developed and the experimental investigations show that the synthesized 3-DOF CPM can achieved a large workspace of 8°×8°×5.5 mm, high stiffness ratios, i.e., >200 for non-actuating over actuating stiffness, and a measured natural frequency of 84.4 Hz.



## 1. Introduction

Compliant mechanism has been a popular solution for developing positioning stages in precise motion systems. Depending on the merits of elastic deformation such as zero backlash, frictionless as well as maintenance-free [1], the compliant mechanisms provide highly repeatable motions that the traditional ball-bearing based counterparts fail to deliver. Compliant mechanisms can be classified into two types, i.e., serial and parallel designs. For precise motion systems, the compliant parallel mechanisms (CPMs) are preferred because closed-loop parallel architecture offers symmetrical configuration that is less sensitive to external mechanical disturbances, higher payload but limited stroke. In addition, compliant joint/limb that delivers larger displacement exhibits poor off-axis (non-actuating direction) stiffness and such poor stiffness characteristic also leads to low dynamic behavior. As a result, developing a multiple degrees-of-freedom (DOF) spatial-motion CPM that offers high stiffness ratios, large workspace, and fast dynamic response, remains as one of the major challenges in the area of compliant mechanism.

Over the past one decade, many methods were proposed to design the CPMs and each has exhibited some benefits as well as limitations. The traditional method synthesized CPMs uses known parallel-kinematic configurations and articulated them via the flexure joints. Various CPMs have been synthesized by this method, ranging from simple 1-DOF [2,3], 2-DOF [4–8] and 3-DOF CPMs [9–12] to more complex 5-DOF [13] and 6-DOF CPMs [14,15]. However, the traditional method depends heavily on human's intuition to synthesize a CPM. To balance the stiffness ratio, i.e., the non-actuating stiffness over the actuating stiffness, and desired workspace has proven to be a challenging process without even considering the desired dynamic property of the CPMs. Recently, the topology optimization method, which optimizes the stiffness and dynamic performance of a CPM using a novel mapping technique, was introduced [16]. It is a holistic design approach that accounts for the stiffness, mass distribution (affects the dynamic), workspace, and size of the CPM during the optimization process [17]. Nevertheless, CPMs with spatial motions cannot be synthesized using this method due to the restrictions of the mapping technique and the inherent 2D design domain. Furthermore, the fitness (objective) function for the stiffness optimization process

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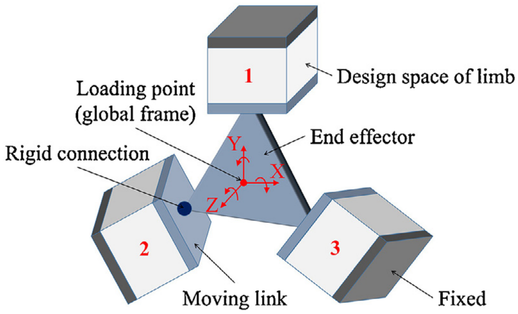


**Fig. 1.** A generalized three-limb CPM with an end-effector fixed to the ground via three symmetrical cubic design spaces.

was not well-defined because the difference in the units, e.g., N/m or N m/rad, etc., for every component within the stiffness matrix was not addressed.

To overcome the limitations of the existing methods, this paper presents a new structural optimization method, termed as the beam-based structural optimization approach. The proposed method is able to optimize both the stiffness and dynamic properties of the CPMs with spatial motions. In addition, a new analytical approach is introduced to normalize the difference in the units for every component within the stiffness matrix in order to derive well-defined fitness functions for the stiffness optimization process. Subsequently, the dynamic optimization process is carried out to achieve the desired resonant frequency while keeping the stiffness performance as high as possible. In this work, the effectiveness of the proposed method is demonstrated through the synthesis and experimental evaluation of a 3-DOF CPM with a out-of-plane motion. The remaining of the paper is organized as follows: the concept and modeling of the proposed method is described in details in Section 2 while the design and experimental investigations of the 3-DOF ($\theta_x - \theta_y - Z$) CPM are discussed in Sections 3 and 4 respectively.

## 2. Beam-based structural optimization approach

### 2.1. Principle

A CPM consists of a moving platform (end effector) that is connected by several compliant limbs as frictionless support bearings. To synthesize multi-DOF spatial-motion CPMs, a novel beam-based structural optimization approach is proposed to optimize the structure of each limb. By taking the illustrated 3-limb CPM in Fig. 1 as an example, the end effector is considered as a solid platform and the three compliant limbs are rotational symmetric about the center. The design space of each limb is a cube with one end being a moving link, which is connected to the end effector, while the other is fixed to the base. Within the cubic design space, a curved-and-twisted (C-T) beam as shown in Fig. 2a is proposed as the initial architecture of each limb. As the proposed method is to be applicable for synthesizing CPMs with up to 6 DOF, each limb must provide 6 DOF, which are realized from the elastic deformation of the C-T beam.

Here, Bezier-curve is used to generate the possible solutions for compliant limbs based on the desired motions. Although this technique has the ability to synthesize various topologies, such topologies are limited to planar motions [18]. In this work, a start twist angle and an end twist angle are added to this technique so that the orientation as well as the twist property of the C-T beam can be determined. Consequently, 6 DOF can be achieved at its free end. Subsequently, special geometries can be generated from such a C-T beam by sweeping a thin rectangular cross-sectional area through a cubic Bezier-curve together with a change in orientation at both ends to create the twist property.

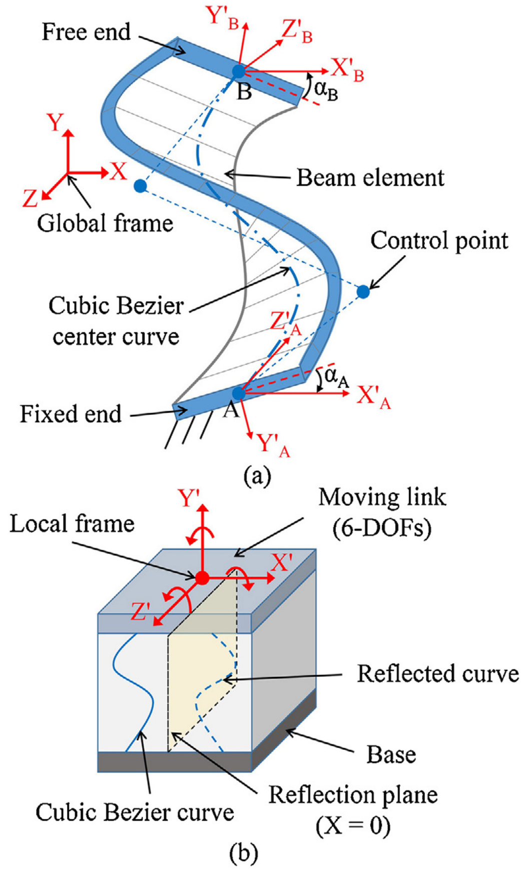


**Fig. 2.** Construction of (a) C-T beam and (b) compliant limb.

In addition, Bezier-curve with twist angles can be changed into any form of flexure including the traditional straight beam-like shape, which is barrier for other curve architectures such as helix and spiral. As illustrated in Fig. 2a, the straight profile can be obtained when all control points of the Bezier-curve locate on a line and when the difference of twist angle between two ends is zero, the flat geometry can be obtained. Referring to Fig. 2b, a pair of symmetrical C-T beams about the $Y'Z'$ ($X' = 0$) plane are used to construct the structure of the compliant limb of the CPM. First, a cubic Bezier-curve is defined followed by another reflected curve mirrored from it. Subsequently, both C-T beams are obtained by sweeping a rectangular cross-sectional area through these curves. The sweep operation is carried out from the start point ($A$) to the end point ($B$) of the Bezier-curve. Referring to Fig. 2a, the local frame at any position on the C-T beam is defined as follows: the $Z'$-axis of the local frame is coincident with the tangent vector of the Bezier-curve and the local $X'$-axis is defined by the projection of the global $X$-axis onto the local $X'Y'$ plane. The orientation of the cross-sectional area at a specific position on the C-T beam on the $X'Y'$ plane is defined by the twist angle. The twist angle is the angle between the $X'$-axis and the long edge of the rectangular cross-sectional area. For examples, the twist angle at point $A$ and $B$ are defined by $\alpha_A$ and $\alpha_B$ as shown in Fig. 2a.

Fig. 3 shows the flow of the structural optimization [17] that is employed in the proposed beam-based approach. The desired DOF and specifications such as the boundary dimensions and the initial cross-sectional area of the C-T beams etc., must be first specified. This is followed by defining the geometrical design variables of the compliant limb structure for the stiffness optimization. The stiffness optimization is carried out by finding the optimized fitness function, $f$, expressed in Eq. ((10)). In this process, the