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A solution of worst-case tolerance analysis for partial parallel chains based on the Unified Jacobian-Torsor model

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ABSTRACT

The Unified Jacobian-Torsor model is highly suitable for tolerance analysis of complex assemblies that are constituted by a large number of rigid parts. However, partial parallel chains with leverage effect of geometrical structures widely exist in these assemblies, especially in a transmission device, such as gear-box; but this model only considers the serial kinematic chains in the assemblies. If this type of partial parallel chains was ignored, it would lead to a large loss in accuracy and reliability; few previous studies have focused on it. In this paper, a solution of worst-case tolerance analysis based on Unified Jacobian-Torsor model is proposed to solve this type of partial parallel chains. First, the parallel joints that form parallel chains in the assembly are determined. Second, the contact points originated in the CLIC (localization tolerancing with contact influence) method are used to determine the boundary position of the parallel joints, and the analysis points from the analysis line method are determined to calculate their deviations at each joint. Third, these deviations are used together to calculate the torsor parameter of the parallel joints according to their torsor features and geometric structure. Fourth, the partial parallel chains are replaced by a new serial chains with the obtained torsor parameters to calculate the final deviations of the functional requirements based on the Unified Jacobian-Torsor model. An industrial application of the spiral bevel gear-box is given to demonstrate this solution.

1. Introduction

Tolerance analysis model is used to predict the quality of an assembly when the tolerances have been assigned to the components. Proper component tolerances can reduce production costs and increase quality. The analysis results are valuable references for tolerances assignment and optimization, and they are also quite useful for industries to produce high quality assemblies at lower cost. In order to obtain the analysis results, mathematical tolerance analysis models should be adopted.

Due to the advantages of clear geometric concept, simple expression form and convenient algebraic operation, screw-theory-based tolerance analysis models are widely used for tolerance analysis in factories, such as the torsor model, Unified Jacobian-Torsor model [1] and matrix model. By using six small displacement torsor, the screw theory proposed by Clément [2,3] was applied to the tolerance analysis. The foremost works in screw theory that used screw parameters to model three dimensional (3D) tolerance zones were proposed by Desrochers [4] and Legoff, Villeneuve [5]. The Jacobian model is quite suitable for deviation propagation. This model was used for tolerance analysis of rigid assemblies with geometric tolerance by Polini and Corrado [6]. In order to overcome the disadvantages of screw theory that is difficult to represent the deviation propagation, the Jacobian model has been integrated into Unified Jacobian-Torsor model which was presented by Desrochers, Ghie [7] and it has been developed for deterministic computer aided-tolerancing [8,9] and statistical tolerance analysis [10,11].

Therefore, Unified Jacobian-Torsor model is quite suitable for tolerance analysis of complex assemblies. However, this model only considers the serial kinematic chains in assemblies. Several reasons may account for this problem. First, the Jacobian matrix for a parallel structure is quite complex [12]. Second, the torsor model, which represents the tolerance of parallel connections, is not well developed [13]. Third, parallel connections form partial parallel chains and there are many different types of parallel connections. Different types

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may need different solutions and it may be the main barrier in solving the partial parallel chains. For example, the torsor model, which represents the tolerance of parallel connections presented in [12], can be obtained by algebraic operations of screw parameters, but it is not suitable for the parallel connections in which screw parameters are influenced by geometrical structures. This is the leverage effect of geometrical structures. This paper is mainly address the solution of this type of parallel structure.

Parallel connections with the leverage effect of geometrical structures widely exist in many complex assemblies, especially in transmission devices such as gear-box. Currently, there are many studies regarding the tolerance analysis of this assembly. The matrix approach proposed by Desrochers and Rivière [14] was applied to a simplified gear pump assembly for the computation of the parallelism error between gears. Another study on the statistical tolerance analysis of bevel gear by tooth contact analysis and Monte Carlo simulation was presented by Bruyere, Dantan [15], but it rarely involved the solution of partial parallel chains. The CLIC [16] and analysis line method [17] were used to calculate the functional requirement of the mechanism that incorporated parallel connections [18]. However, the calculation results only had the position deviation of the translation parameters (u, v, w) without the rotation parameters $\left(\alpha, \beta, \delta\right)$ [19]. Thus, it is not suitable for the mechanism with functional requirement of orientation deviation, such as the complex assembly of a spiral bevel gear-box.

Many three dimensional (3D) tolerance analysis methods have been presented [20–23], and they are suitable for tolerance analysis of these complex assemblies. The research hotspots mainly focus on four typical methods: T-Map (Tolerance-Map) model, Matrix model, Unified Jacobian-Torsor model and DLM (Direct Linearization Method) [21]. The T-Map model proposed by Davidson, Mujezinovic [24] was applied to round face for geometric tolerance. The displacement matrix was used to represent any roto-translational variation of a feature within the tolerance zone and clearance between two features in the matrix model [14]. The Unified Jacobian-Torsor model has been introduced previously. The DLM proposed by Chase et al. [25,26] was presented for tolerance analysis of the assemblies, and this method generalized the vector loop-based model to include small kinematic adjustments based on first order Taylor's series expansion. Several studies have been carried out recently based on these methods. A small displacement torsor model for 3D tolerance analysis of conical structures was presented by Jin, Chen [27]. T-Map model was used to model tolerance accumulation in parallel assemblies for calculating the final functional target feature by Jaishankar, Davidson [28]. A dimensional hierarchization matrix, a tolerance optimization algorithm and 3D CAT software were combined to study for the tolerance analysis and optimization of assembly by Barbero, Azcona [29].

However, the above four typical methods are not all suitable to solve partial parallel chains. The purpose of this paper is to introduce a new solution to solve the partial parallel chains formed by parallel connections with the leverage effect of geometrical structures for Unified Jacobian-Torsor model. Based on this solution, the Unified Jacobian-Torsor model will be suitable for tolerance analysis of mechanism with this type of parallel chains. The advantages of the CLIC method and analysis line method will be integrated to generate a new solution and calculate the six degrees of freedom: translation (u, v, w) and rotation $\left(\alpha, \beta, \delta\right)$ of the parallel chains. The obtained torsor parameters will be integrated into the Unified Jacobian-Torsor model to calculate the final deviations of the functional requirements using the new serial kinematic chains.

The rest of this paper is organized as follows:the Unified Jacobian-Torsor model is introduced in Section 2. A detailed description of partial parallel chains is given in Section 3. The solution to the partial parallel connections with the leverage effect of geometrical structures is discussed in Section 4. An industrial application of a spiral bevel gear-box is given as a demonstration in Section 5, and conclusion is drawn in Section 6.

2. Unified Jacobian-Torsor model

The Unified Jacobian-Torsor model is an innovative 3D tolerance analysis method that combines the advantages of two models: the torsor model and the jacobian's matrix model. The torsor model uses screw parameters including three translational vectors and three rotational vectors to represent small displacements of a feature within its tolerance zone [12,30]. Taking a cylinder feature as an example, L_1 is the nominal axis of the cylinder feature and L_2 is the actual axis caused by tolerance t as shown in Fig. 1. The torsor model which represents the position and orientation of L_2 in relation to L_1 can be expressed as:

$$T_{2/1} = \begin{bmatrix} u & v & w & \alpha & \beta & \delta \end{bmatrix}^T \tag{1}$$

where u, v and w are the three translational vectors along the axes x, y and z, respectively; α, β, δ are the three rotational vectors around the axes x, y and z, respectively.

The Jacobian matrix [9] is used for tolerance propagation in the assembly. In this model, the relationship between the small displacement of all the functional elements (*FE*) and the functional requirement (*FR*) is expressed by the Jacobian matrix and formulated by [7]:

$$\begin{bmatrix} J_1 & \cdots & J_6 \end{bmatrix} = \begin{bmatrix} \left[R_0^i\right]_{3\times3} & \vdots & \left[W_i^n\right]_{3\times3}\cdot\left[R_0^i\right]_{3\times3} \\ \ldots & \vdots & \ldots \\ [0]_{3\times3} & \vdots & \left[R_0^i\right]_{3\times3} \end{bmatrix} \tag{2}$$

where $\mathrm{R}_0^{\mathrm{i}} = \begin{bmatrix} \vec{\mathrm{C}}_{1\mathrm{i}} & \vec{\mathrm{C}}_{2\mathrm{i}} & \vec{\mathrm{C}}_{3\mathrm{i}} \end{bmatrix}$. These vectors represent the orientation of the reference frame i with respect to 0, where the columns $\vec{\mathrm{C}}_{1\mathrm{i}}$, $\vec{\mathrm{C}}_{2\mathrm{i}}$ and $\vec{\mathrm{C}}_{3\mathrm{i}}$ respectively indicate the unit vectors along the axes X_i, Y_i and Z_i of reference mark i in reference mark 0. $\left[W_i^n\right]_{3\times3}$ is a skew-symmetric matrix allowing the representation of the vector $\left[\vec{d}_n - \vec{\mathrm{d}}_{\mathrm{i}}\right]$ with $dx_i^n = dx_n - dx_i$, $dy_i^n = dy_n - dy_i$ and $dz_i^n = dz_n - dz_i$. $\vec{\mathrm{d}}_{\mathrm{i}} = \begin{bmatrix} dx_i & dy_i & dz_i \end{bmatrix}$ is the position vector which defines the origin for the reference frame i in 0. The first three elements in the fourth, fifth and sixth columns