Precision Engineering 47 (2017) 344–352

Contents lists available at ScienceDirect

Precision Engineering

journal homepage: www.elsevier.com/locate/precision

Profile error evaluation of free-form surface using sequential quadratic programming algorithm

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ARTICLE INFO

Article history:
Received 23 May 2016
Received in revised form 29 August 2016
Accepted 19 September 2016
Available online 21 September 2016

Keywords:
Free-form surface profile error
Signed distance function
Surface subdivision
STL model
Sequential quadratic programming

ABSTRACT

Profile error of free-form surface is evaluated in this paper based on sequential quadratic programming (SQP) algorithm. The optimal localization model is established with the minimum zone criterion firstly. Subsequently, the surface subdivision method or STL (STeror Lithography) model is used to compute the point-to-surface distance and the approximate linear differential movement model of signed distance is deduced to simplify the updating process of alignment parameters. Finally, the optimization model on profile error evaluation of free-form surface is solved with SQP algorithm. Simulation examples indicate that the results acquired by SQP method are closer to the ideal results than the other algorithms in the problem of solving transformation parameters. In addition, real part experiments show that the maximum distance between the measurement points and their corresponding closest points on the design model is shorter by using SQP-based algorithm. Lastly, the results obtained in the experiment of the workpiece with S form illustrate that the SQP-based profile error evaluation algorithm can dramatically reduce the iterations and keep the precision of result simultaneously. Furthermore, a simulation is conducted to test the robustness of the proposed method. In a word, this study purposes a new algorithm which is of high accuracy and less time-consuming.

1. Introduction

Free-form surfaces have been widely used in many fields such as aerospace, automobile, shipbuilding, mold and many other industrial fields due to their superior geometric features. However, the surface profile error is led by various factors, such as cutter displacements [1] and tool deflection [2,3], which may affect the performance of free-form surface inevitably. Therefore, the processing quality of free-form surfaces becomes increasingly significant for normal operation of the whole system. Demands have been rising on the quality manufacturing of parts with complex surfaces, which ultimately require precise measurement and inspection [4]. According to ISO1101 [5], the form error of parts should be within the range of the minimum zone. However, the widely used least-square method does not meet the standard, resulting in inaccuracy of the inspection result. In light of this fact, further study is necessary to improve the accuracy of the inspection result. During the past decades, many researchers focused on the profile error evaluation of free-form surface. In terms of inspection, how to calculate point-to-surface distance and realize localization remain the critical problems. On the one hand, the point-to-surface distance needs to be calculated by choosing the closest points from the design surface. The iterative closest points (ICP) algorithm is usually used to find the closest points between points on two surfaces. Besl and Mckay [6] applied an iterative closest point (ICP) process to search the corresponding point of the measurement point in 1992. Zhang [7] solved the geometric matching problem of two free-from curves by combining ICP algorithm with the least-squares technique in 1994, but the initial iteration arrangement is required rigorously for the ICP algorithm. Considering ICP based on closest points only exhibits local linear convergence, Pottmann et al. [8] used quadratically convergent algorithm to promote the application of ICP in 2006. Schweinoch et al. [9] proposed a non-registration method to further optimize ICP efficiency and application. Aiming at weakening the affection of initial arrangement to improve robustness of ICP algorithm, Li and Yin [10] defined a new point-surface distance function (ADF) which increases ICP convergent speed obviously in 2010. Based on work [10], Li and Yin considered the affection of various curvatures of measured points and defined modified coefficient in ADF to get better approximation of the point-to-surface closest distance in [11]. Some else methods have been proposed besides ICP algorithm.

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http://dx.doi.org/10.1016/j.precisioneng.2016.09.008
0141-6359/

Ullrich et al. [12] focused on computation of Euclidean distance between an arbitrary point and a surface by surface subdivision. Liu [13] calculated surface profile error based on linear quadtree division of surface grid after rough localization. Zhu et al. [14,15] defined a signed point-to-surface distance function and derived its differential motion model, and obtained the closest point set by adjusting the orientation of the measured 3-D coordinate data. On the other hand, for localization, based on the Gaussian and the mean curvatures as object matching features, Ko KH [16] established a non-linear polynomial equation system which was solved with the Interval Projected Polyhedron (IPP) algorithm in 2003. Lai and Chen [17] proposed an algorithm which iterates CMM measurement and coordinate translation for parts that have no regular feature to be referenced. With the development of optimization technology and methods, some heuristic search algorithms such as genetic algorithm (GA) [18–20], particle swarm optimization (PSO) [21–23] et.al, are applied to solve the localization problem. Due to their own limitations, these methods are unlikely to provide an optimal solution universally to some extent. Currently, surface description methods such as Coon, Bezier, B-spline, and NURBS are adopted to describe the free-form surface. After fitting the set of measurement points by a bi-cubic NURBS surface and calculating the differential geometric information of the fitted surface and the design surface respectively, Li and Gu [24] started the general localization based on the corresponding features extracted from both surfaces in 2006. Additionally, the fine localization was implemented to solve the point-to-point correspondence and localize the surface accurately. [25] Sun et al. introduced a unified algorithm in 2009 for workpiece localization and quality evaluation with the method of SQP, and used differential geometric method to search for closest points. Differential geometric method is rigorous to the initial iteration point, but it is hard to carry out in terms of the surface whose parameters are insufficient. He and Liu [26] adopted the same algorithm to evaluate conicity error in 2013. Moreover, He and Zhang [27] developed a method combining Differential Evolution (DE) algorithm and Nelder-Mead (NM) algorithm for evaluating free-form surface profile error in 2015.

The key problems to be solved in profile error evaluation of free-form surface are locating and searching the closet points on the design model corresponding to the measurement points. In the process of optimization alignment for location, computation of point-to-surface distances is involved. In spite of the extensive application of the methods for profile error estimation in free-form surface, some disadvantages cannot be neglected. The problem includes rigorous dependence on initial arrangement for ICP, tedious for some improved ICP and slow convergence speed and prematurity for heuristic search algorithms such as GA and PSO, and so on. Thus, an algorithm which has the property of fast and stable convergence and robustness is proposed in this paper. In Ref. [25] SQP algorithm demonstrate its advantage mentioned above when it is used on the problem of free-form surface profile error evaluation. However, the point-to-surface distance computation method proposed in Ref. [25] needs to solve non-linear equations and set good initial value for the iteration computation. This method is normally time-efficient, but it may not fit all conditions, such as the calculation for some boundary points. Thus, surface subdivision method is used to search the closest points on the surface. For the surfaces which are not defined by parametric equations or offer little geometric information, the algorithm for the calculation of point-to-surface distance is proposed based on the STL model. The combining of SQP method and the distance computation method which is proposed in this paper is more effective to evaluate profile error than the previous contributions.

Therefore, in this paper, a global optimization algorithm is proposed to evaluate the free-form surface profile error based on SQP algorithm. Subdivision method is adopted to calculate the point-to-surface distance for parametric surfaces. For surfaces with insufficient geometric information, the distance from the spatial points to STL model is taken as the distance from those points to design surface. The paper is organized as follows: the design of free-form surface is formulated in NURBS and the point-to-surface distance is elaborated with two different methods. Subsequently, the mathematical model of profile error evaluation is proposed, and the simplified distance function as well as its differential motion expression is developed to solve the min-max optimal problem with the SQP algorithm. Finally, simulation examples and actual experiments are presented to valid the proposed approach.

2. NURBS for describing free-form surface

In the application of computers, free-form surface is always presented by the coordinates and control parameters of discrete points. Most of the three-dimensional software adopts NURBS as an excellent approach to model a free-form surface. NURBS is a common mathematic form used in computer graphics for representing and designing both curves and surfaces [28]. A NURBS surface has the characteristics of local support, affine invariance, strong convex hull, variation diminishing, parameter sequence, local approximation and so on, thus it is widely recognized and used in the CAD/CAM and graphics community. A NURBS surface is the rational generalization of the tensor-product non-rational B-spline, and is defined as following equations [29]:

Let

$$R_{i,j}(u,v)=\frac{N_{i,p}(u)N_{j,q}(v)\omega_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m}\omega_{i,j}N_{i,p}(u)N_{j,q}(v)},\ 0\le u,v\le 1 \tag{1}$$

so

$$S(u,v)=\frac{\sum_{i=0}^{n}\sum_{j=0}^{m}\omega_{i,j}P_{i,j}N_{i,p}(u)N_{j,q}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m}\omega_{i,j}N_{i,p}(u)N_{j,q}(v)}=\sum_{i=0}^{n}\sum_{j=0}^{m}R_{i,j}(u,v)P_{i,j} \tag{2}$$

where $\omega_{i,j}$ are the weights, $P_{i,j}$ form a control net, and $N_{i,p}(u)$, $N_{j,q}(v)$ are the normalized B-splines of degree p and q in the u and v directions, respectively, defined over the knot vectors described by equation:

$$\begin{cases}\mathbf{U}=\left[0=u_0=\cdots=u_p,u_{p+1},\ldots,u_{r-p-1},u_{r-p}=\cdots=u_r=1\right]\\ \boldsymbol{V}=\left[0=v_0=\cdots=v_q,v_{q+1},\ldots,v_{s-q-1},v_{s-q}=\cdots=v_s=1\right]\end{cases}, \tag{3}$$

where the home knots and the end knots are repeated with multiplicities $p+1$and$q+1$, respectively, and $r=n+p+1$,$s=m+q+1$.

3. Point-to –surface distance

3.1. Surface subdivision method to search the closest point

Given the manufactured errors and the random errors in measurement, the measurement point cannot be fully corresponded to the theoretical point on design surface so that it is necessary to obtain the closest points on the design surface. By adopting surface subdivision method or differential geometric method, the closest point is easy to be found. Surface subdivision method is hence adopted here for its simple principle and convenient operation. The parametric equation of surface$\mathbf{r}(u,v)$ is assumed as$x=$