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Selection of fiducial locations and performance metrics for point-based rigid-body registration

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ABSTRACT

A method is described to select the location and number of fiducials used in point-based, rigid-body registration of two coordinate frames. Two indices are introduced which are used to search for the optimum configuration of fiducials. They can be used to quickly evaluate a large number of configurations because no actual registration is involved in their calculation. Furthermore, configurations yielding small values of the indices correlate well with configurations which result in optimum registrations. Three registration performance metrics are discussed, and it is shown that optimization of different metrics leads to different selection of fiducial configurations. If an optimized configuration is selected as a starting configuration of N fiducials, the addition of extra fiducials does not significantly improve the registration in most cases. This work is based on 3D data acquired with three different instruments, each having different noise and bias characteristics.

1. Introduction

The aim of a registration procedure is to obtain a transformation between two coordinate frames. Usually, sensors acquire the location of a point in three dimensional (3D) space in their own local coordinate frame. When positional data are obtained by two different sensors or two datasets are acquired with the same instrument placed in two different poses, some of the points may be measured only in one frame but have to be accessed in the other frame. Then, the transformation to map a set of points measured in one frame to the other is needed. The first coordinate frame (from which the data are transformed) will be called the working frame and the second one (to which the data are transformed) the destination frame. In point-based, rigid-body registration, the parameters of the transformation are determined using measurements of the same physical points acquired in both frames. These common points form a list of N pairs of corresponding points called fiducials. In the ideal case when the measurement of the fiducials is noise and bias free, the rigid-body assumption dictates that the distance between any two fiducials in the working frame is equal to the distance of the corresponding two points in the destination frame. In reality, every consecutive i-th measurement of the same points yields a slightly different pair of datasets: $\{\boldsymbol{X}\}_{N,i}$ in the working and $\{\boldsymbol{Y}\}_{N,i}$ in the destination frames. Then, the transformation is the rotation $\boldsymbol{R}_i$ and translation $\boldsymbol{\tau}_i$ which minimize the Fiducial Registration Error, FRE_i

$$FRE_i\left(\{\boldsymbol{X}\}_{N,i},\{\boldsymbol{Y}\}_{N,i}\right)=min\left(\frac{1}{N}\sum_{n=1}^{N}\|\boldsymbol{R}_i\boldsymbol{X}_{n,i}+\boldsymbol{\tau}_i-\boldsymbol{Y}_{n,i}\|^2\right) \quad (1)$$

One may be tempted to use FRE_i as a metric for the quality of registration. However, in the general problem of fitting a model to noisy data, a large residual value of the error function may be obtained in two different situations: 1) correct model is fitted to data with large noise; 2) wrong model is fitted to data with small noise. Therefore, a better metric is needed to quantify the performance of registration. Once the registration transformation $(\boldsymbol{R}_i, \boldsymbol{\tau}_i)$ is determined, it can be applied to a target point which is not a fiducial (i.e., not used to calculate the registration transformation) and the Target Registration Error $TRE_i(\boldsymbol{T_x})$ is defined as:

$$TRE_i(\boldsymbol{T_x})=\|\boldsymbol{R}_i\boldsymbol{T_x}+\boldsymbol{\tau}_i-\boldsymbol{T_y}\|^2, \quad (2)$$

where $\boldsymbol{T_x}$ and $\boldsymbol{T_y}$ are the target positions in the working and destination frames. In general, theoretical formulations of $TRE_i(\boldsymbol{T_x})$ assume no noise or bias in the target measurements and thus, the only source of uncertainty comes from noisy registration $(\boldsymbol{R}_i, \boldsymbol{\tau}_i)$. Non-zero values of FRE_i and TRE_i are a consequence of non-zero

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Fiducial Localization Error FLE_i defined as

$$FLE_i\left(\left\{\boldsymbol{X}\right\}_{N,i},\left\{\boldsymbol{Y}\right\}_{N,i}\right)=\frac{1}{N}\sum_{n=1}^{N}\left(\|\boldsymbol{X}_{n,i}-\boldsymbol{X}_{n,0}\|^2+\|\boldsymbol{Y}_{n,i}-\boldsymbol{Y}_{n,0}\|^2\right),\tag{3}$$

where $\left\{\boldsymbol{X}_0\right\}_N$ and $\left\{\boldsymbol{Y}_0\right\}_N$ are the true unknown locations of the fiducials.

Much effort has been made to formulate an analytical relation between *FLE*, *FRE*, and *TRE*, where

$$FLE=\langle FLE_i\rangle,\ FRE=\langle FRE_i\rangle,\ TRE=\langle TRE_i\rangle,\tag{4}$$

where $\langle\ldots\rangle$ indicates averaging over repeated measurements of fiducials acquired in the same experimental conditions, as defined in [1]. Different analytical formulas have been derived. They depend on different models of Gaussian noise perturbing the true locations of the fiducials in the working and destination frames

$$\boldsymbol{X}_{n,i}=\boldsymbol{X}_{0,n}+\boldsymbol{B}_{\boldsymbol{x},n}+\boldsymbol{\xi}_{n,i},\ \boldsymbol{Y}_{n,i}=\boldsymbol{Y}_{0,n}+\boldsymbol{B}_{\boldsymbol{y},n}+\boldsymbol{\eta}_{n,i},\tag{5}$$

where $\boldsymbol{B}_{\boldsymbol{x},n}$ and $\boldsymbol{B}_{\boldsymbol{y},n}$ are systematic biases at the n-th location while $\boldsymbol{\xi}_{n,i}$ and $\boldsymbol{\eta}_{n,i}$ are random perturbations with zero-mean Gaussian distributions. Historically, the oldest and simplest model used is:

$$cov\left(\boldsymbol{\xi}_n\right)=cov\left(\boldsymbol{\eta}_n\right)=\sigma\boldsymbol{I}_{3\times3},\ \boldsymbol{B}_{\boldsymbol{x},n}=\boldsymbol{B}_{\boldsymbol{y},n}=0\tag{6}$$

for all $n=1, \ldots, N$. This model corresponds to homogenous (i.e., independent of location n), isotropic (covariance matrix has only equal diagonal elements), zero-mean Gaussian noise. Based on this model, two noteworthy equations were derived. First in [2] and then in [3], it was shown that

$$FRE=\left(1-\frac{2}{N}\right)FLE,\tag{7}$$

where $N\geq3$ is the number of fiducials used for registration. This equation has two rather surprising and counterintuitive implications. First, *FRE* increases with increasing N; second, *FRE* does not depend on the spatial distribution of the fiducials. Since noise is assumed to be homogeneous, there is no good or bad geometrical distribution of fiducials (excluding extreme configurations of nearly collinear points). The second noteworthy equation derived in [3] relates *TRE* with *FLE*

$$TRE\left(\boldsymbol{T_x}\right)=FLE\times\left(\frac{1}{N}+\frac{1}{3}\sum_{k=1}^{3}d_k^2/M_k^2\right),\tag{8}$$

where M_k^2 is the moment of inertia of the fiducial configuration about the principal k-th axis (defined by fiducials $\left\{\boldsymbol{X}_0\right\}_N$) and $d_k\left(\boldsymbol{T_x}\right)$ is the distance of the target $\boldsymbol{T_x}$ to the k-th principal axis. A few important conclusions may be derived from (8): 1) contrary to *FRE*, *TRE* depends on the geometrical configuration of the fiducials $\left\{\boldsymbol{X}_0\right\}_N$; 2) a target located close to the centroid of $\left\{\boldsymbol{X}_0\right\}_N$ should have small *TRE*; 3) as N increases *TRE* decreases. Subsequent efforts using more realistic noise models led to further modifications of the closed form equations for *FRE* and *TRE* [4–8]. Practical use of these analytical expressions is limited due to the fact that *FLE* cannot be measured experimentally as it depends on the true locations of the fiducials $\left\{\boldsymbol{X}_0\right\}_N$ and $\left\{\boldsymbol{Y}_0\right\}_N$ which are unknown [9,10]. Attempts to relate $TRE_i\left(\boldsymbol{T_x}\right)$ to measurable FRE_i were hampered by the discovery that TRE_i, $FRE_i=0$, i.e., they are statistically uncorrelated [7]. Furthermore, the method of removing bias from the fiducial measurements requires measurements of targets that are bias free [11] and this restriction cannot be satisfied in many realistic experimental settings. Finally, if $TRE_i\left(\boldsymbol{T_x}\right)$ from (2) is used as a metric for registration performance, then the search for the optimum placement of fiducials may be incorrect because noise and bias in the measurement of target $\boldsymbol{T_x}$ are ignored in (2). Thus, for realistic noise characteristics (anisotropic, heterogeneous, and with non-zero bias), the performance of registration depends on the selection of fiducial locations, and in spite of intensive theoretical efforts, no analytical method exists to guide practitioners in selecting the optimum placement of fiducials.

The problem of finding the best placement of fiducials was studied extensively in the area of image assisted neurosurgery [12–17]. In this paper, we follow the strategy for finding the best fiducial configuration outlined in [18]. The method is based on an exhaustive linear search of all possible combinations of N fiducials from M potential locations. The number of such combinations $M_N=\frac{M!}{N!(M-N)!}$ grows rapidly with M and N (in our experiments $M=125$, $N=4$, and $M_N=9{,}691{,}375$). In principle, for each combination, the corresponding transformation should be calculated first and then applied to the target(s) to gauge the registration quality. However, for a large number of combinations M_N, this approach is time consuming and not practical. Therefore, we propose two proxy indices which do not require the calculation of the transformation matrix and, hence, they both can be quickly determined for a large number of combinations M_N. We show that these indices correlate well with the characteristics of the actual registration. We also show that if a combination of $N=4$ fiducials is optimally selected according to a certain performance metric, then the addition of more fiducials leads to marginal or small improvement in the registration performance. However, this behavior may not hold for a different performance metric and the addition of more fiducials may worsen the performance. The choice of performance metric is a subtle issue which can have far reaching consequences. The metrics used in this study fall in two categories. One category minimizes the uncertainty of a target point $\boldsymbol{T_x}$ transformed into the destination frame, i.e., it minimizes the spread of a point transformed by noisy transformations $(\boldsymbol{R}_i, \boldsymbol{\tau}_i)$ obtained from repeated measurements of the fiducials. The second category minimizes the distance between $\boldsymbol{T_x}$ transformed by $(\boldsymbol{R}_i, \boldsymbol{\tau}_i)$ and the corresponding target $\boldsymbol{T_y}$ in the destination frame. The two categories reflect the dichotomy of each measurement, i.e., its precision and accuracy. Both are desired but practitioners should be aware that optimizing fiducial placement based on one criterion does not necessarily satisfy the second.

We verify our approach using 3D data acquired with three different instruments, each having different noise characteristics: a laser tracker (LT), a motion tracking system (System A), and a large-scale metrology system (System B). For the purposes of this research, LT is considered to have no noise and no bias, System A has small noise and large bias, while System B has large noise and small bias. The three sets of acquired 3D data (each containing repeated measurements of the same points) allowed us to evaluate three registrations (A to LT, B to LT, and A to B) covering a wide spectrum of realistic conditions.

The paper is organized as follows: in Section II, a brief description of experimental set-up and data processing is provided. Also, in this section, definitions of the proxy indices and metrics which are used to gauge the quality of registration are provided. In Section III, the results are presented, followed by a discussion in Section IV, and final conclusions are presented in Section V.

2. Experiment and data processing

Positions of 125 3D points distributed on a semi-regular $5\times5\times5$ grid were measured using three different instruments: System A (motion capture system), System B (large scale metrology), and laser tracker LT [19]. In addition, 16 other points randomly located in the work volume ($3\,\text{m}\times3\,\text{m}\times1.8\,\text{m}$) were also measured by the three instruments. Registrations between three pairs of instruments (A to B, A to LT, and B to LT) were performed for different configurations of N fiducials selected from the 125 grid points. The