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Model selection for degradation modeling and prognosis with health monitoring data

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ABSTRACT

Health monitoring data are increasingly collected and widely used for reliability assessment and lifetime prediction. They not only provide information about degradation state but also could trace failure mechanisms of assets. The selection of a deterioration model that optimally fits in with health monitoring data is an important issue. It can enable a more precise asset health prognostic and help reducing operation and maintenance costs. Therefore, this paper aims to address the problem of degradation model selection including goals, procedure and evaluation criteria. Focusing on continuous degradation modeling including some currently used Lévy processes, the performance of classical and prognostic criteria are discussed through numerous numerical examples. We also investigate in what circumstances which methods perform better than others. The efficiency of a new hybrid criterion is highlighted that allows to take into account the information of goodness-of-fit of observation data when evaluating prognostic measure.

1. Introduction

Degradation modeling in the presence of health monitoring data is extremely important for lifetime prognosis and maintenance planning. Complex models permit to take advantage of all available information and describe precisely the dynamics of degradation. However, these models are not easily tractable, and their calibration in the presence of data is a burdensome task. On the other hand, a very simple model which can be easily fitted to data but can underestimate or overstate the uncertainty around the lifetime prediction. This latter can induce risks and additional costs in prognosis based decision making and maintenance. A useful and suitable degradation model leads to a balance between accuracy and tractability, [1,2].

The degradation considered as a random phenomenon often has a gradual time-continuous trajectory. Regarding the system under consideration, the degradation can take values in discrete or continuous space. For instance, in a crack growth phenomenon, the crack length can take infinite possible values as soon as it begins to grow. Similarly, a deteriorating production process can have several quality states which will impact the production and result of gain or losses. In these two cases, the modeling procedure should take into account the phenomenon under consideration, see for instance [3–5].

This paper focuses on the gradual degradation modeling and prognosis with health monitoring data. When data is available, the important issue is to select the model which describes the underlying degradation phenomenon in the best possible way. The data are collected under given environmental conditions and may not represent the average behavior of the deteriorating system. A suitable model is one who can take into account the possibility of extreme behaviors during data collection without losing in perspective the real average degradation behavior. The final use of the degradation model can largely impact the way data is handled, and a model is favored. If the result of modeling has a large impact on safety issues, the choice and the procedure are not carried out in the same way as if only economic profits or losses are taken into account. Similarly, the cost induced by decision policies based on different models may influence the model selection. Regarding statistical properties, the best candidate is well characterized from data trajectories. It can permit fast calibration and straightforward lifetime estimation. For more details and examples, refer to [6,7].

To be able to discard irrelevant models it is necessary to have some prior knowledge about the degradation phenomenon under consideration. In the presence of such information and degradation data, the goal of degradation modeling is to select one model from a set of competing models that best captures the underlying degradation process. As it is mentioned before the selection criteria depend mainly on the specific purpose for which the model can be used, see for instance [5,8].

This paper deals partly with Lévy processes [9] and focuses on the most commonly used which are Wiener and Gamma processes. Model calibration and data fitting of these models have been widely addressed

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in the fields of finance, biology and engineering [10,11]. However, the model selection criteria in the finance and biology field have some differences with engineering domain where maintenance and safety constraints are significant concerns [5,12,13]. The model selection for engineering prediction problems is an important issue but has not been generalized and extensively addressed, [14].

This paper proposes different criteria for model selection, to avoid system failure and with the most reasonable calculation time. First, an overview of the considered stochastic processes is given, and their use in degradation modeling is underlined. Afterwards, to provide a first selection criterion and to continue to outline a methodological guide, some widely addressed and well known statistical data fitting criteria are pointed out. Their limits and performances are highlighted, and new prognostic basis criteria are introduced. To complete the methodological task, the proposed procedure is applied to different simulated data sets. The behavior, weakness, and performances of the model selection are analyzed and discussed.

The remainder of the paper is as follows. Section 2 describes the set of models under consideration. In Section 3, the criteria for the data-based model selection are exposed. Section 4 gives some prognosis criteria. Eventually, in Section 5 the proposed models and criteria are tested on simulated data.

2. Stochastic degradation modeling and parameter estimation

This section focuses on the considered Levy processes in the framework of degradation modeling, [12,15,16]. A Lévy process is a stochastically continuous process with stationary and independent increments. It can be decomposed into the sum of a drifted Brownian motion and a jump process such as Gamma or Poisson process. These properties make this class of processes a good candidate for degradation modeling. In this paper, the widely used Levy processes, particularly Brownian and Gamma family, are the models under consideration to describe the degradation behavior. They are presented in the following.

Let us introduce some notations. Let X_t be the degradation level of the system at time t. Let L be the failure threshold in the sense that the system is supposed to be failed if the degradation level exceeds the level L. Let $t_{ps}(L)$ be the first passage time of the degradation process to level L:

$$t_{ps}(L) = \inf\{t \in \mathbb{R}^+, X_t \geq L\}. \tag{1}$$

The residual useful life time (RUL) at time t given $X_t = x$ denoted by $RUL_{(x,t)}$ is the time duration before the first passage time $t_{ps}(L)$ starting from the degradation level x at t. In the following $t_{ps}(L)$ will be denoted t_{ps} except in case of ambiguity.

2.1. Lévy type diffusion process

To consider a general degradation modeling framework and take into account the possible existing physical models, it is natural to introduce stochastic differential equations (SDE) based on a standard Brownian motions B_t:

$$dX_t = m(X_t, t)dt + \sigma(X_t, t)dB_t,$$

$(m, \sigma) : \mathbb{R} \times \mathbb{R}^+ \mapsto \mathbb{R}$ are respectively the drift and the diffusion coefficient. These equations appear at the beginning of 20th century in statistical mechanics and have been thoroughly formulated by Itô [17,18]. Such equations can be derived directly from existing physical models by adding Gaussian "white noises" on measurements. They permit a wide range of degradation modeling due to the flexibility of the structure and functional parameters.

In this section, some specific Levy diffusions processes are presented, and their interest in degradation modeling is underlined. For each case, the differential equation, the related distribution functions, and some statistical properties are exposed.

2.1.1. Wiener process

The Wiener process is very popular deterioration modeling when observations increments vary non-monotonically. The statistical properties of the failure time in the case of a Wiener process are studied in [19,20]. It has been considered in reliability and lifetime analysis widely since the 1970s. Authors in [21] used the Wiener process with drift to model accelerated life testing data. In [22,23] the impact of measurement errors on the Wiener degradation model of self-regulating heating cables is analyzed. Authors in [24–26] also focused on the stopping time (failure time) of Wiener deterioration models and expanded the existing theoretical results in this domain. The Brownian motion with non-linear drift has attracted more attention in engineering problems and residual lifetime estimation, see for example [27–29].

More precisely, a diffusion process in Brownian motion family has the following properties. The increments are independent, X_t is solution of the SDE $dX_t = \mu(t)dt + \sigma dB_t$, where $\mu(t)$ is a function of t and B_t is a standard Brownian motion. The transition probability to $X_t = x$ knowing that $X_s = y$ is given by:

$$p(x, t|y, s) = \frac{1}{\sqrt{4\pi\sigma(t-s)}} exp\left(-\frac{(x + M(t,s) - y)^2}{4\sigma^2(t-s)}\right), \tag{2}$$

where $M(t, s)$, $\gamma(t, s)$ are given by:

$$M(t, s) = -\int_s^t \mu(u)du, \tag{3}$$

The mean and variance values of X_t are given by:

$$\mathbb{E}[X_t] = -M(t, 0), \mathrm{Var}[X_t] = \sigma^2 t \tag{4}$$

M_1*: Wiener process with linear drift.* This process is the special case of a Wiener process when the drift and the variance are not time dependent ($\mu(t) = \mu$ is constant). This diffusion process which is also a Lévy process is suitable for fluctuating degradation records linearly increasing in time. It will be referred as M_1 in Section 5.

The RUL cumulative distribution function (cdf) for a drifted Brownian motion given the observation value $X_t = x$ at the observation time t are given as follows [30]:

$$F_{RUL_{(x,t)}}(u) = \Phi\left(\frac{-L + \mu u + x}{\sigma\sqrt{u}}\right) + e^{2\frac{\mu}{\sigma^2}(L-x)}\Phi\left(\frac{-L - \mu u + x}{\sigma\sqrt{u}}\right) \tag{5}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function.

M_2*: Wiener process with time-dependent drift.* This process is the particular case of a diffusion process when the degradation process is exponentially increasing in time ($\mu(t) = at^b$). It will be referred as M_2 in Section 5. In this case, the ratio between its drift and diffusion is not a constant and also depends on the time. Therefore, it is difficult to derive the explicit expression of the RUL distribution. Its evaluation requires solving a non-singular Volterra Integral Equation. It can be done numerically, see e.g. [31].

2.1.2. M_3: diffusion process with purely time-dependent drift and diffusion

This process is the particular case of a diffusion process when $\mu(t)$ and $\sigma(t)$ are time dependent functions independent of X_t). This is suitable for a degradation process including random walks with time-dependent drift and diffusion terms. In this paper, we consider a special case of the purely time dependent drift and diffusion Brownian motion: $\mu(t) = cat^b$ and $\sigma(t) = \sqrt{2at^b}$. It will be denoted as M_3 in Section 5. As the power-law drift is proportional with drift, according to [32], the RUL CDF of the process at time t given a degradation level at time t, $X_t = x$, is derived:

$$F_{RUL_{(x,t)}}(u) = \Phi\left(\frac{-L - c\gamma(t_{ps}, t) + x}{\sqrt{2\gamma(t_{ps}, t)}}\right) + e^{c(L-x)}\Phi\left(\frac{-L - c\gamma(t_{ps}, t) + x}{\sqrt{2\gamma(t_{ps}, t)}}\right) \tag{6}$$

where $\gamma(t_{ps}, t)$ is given by Eq. (3).