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Optimal allocation of units in sequential probability series systems☆

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ABSTRACT

A concept of Sequential Probability Series System (SPSS) is developed in this paper, which widely exists in many practical sectors such as power plants, inventory management and security management. In SPSS the failure states of each unit are divided into two classes according to their consequences: dangerous failure and safe failure, where the former results in system failure while the latter has no impact on the system. Suppose that when a failure unit appears in SPSS, the system fails with probability p while the other units in SPSS can continue working with probability $1-p$. This paper treats the problem of achieving optimal allocation of units in SPSSs that maximizes expected total working time of all units. Three optimal allocation models are formulated. We derive the analytical expressions for the optimal allocation solutions under certain assumptions. A genetic algorithm and a Monte Carlo method are provided to solve the allocation problems whose analytical solutions are difficult to obtain. An application can be found in Remote Power Feeding System (RPFS). Numerical examples for a RPFS are presented to demonstrate the application of the developed approach in each model.

1. Introduction

Many researchers have investigated the optimal allocation of units in reliability systems, because the optimal allocation problems are the most important in designs of systems and products, which determine the intrinsic reliability of the systems and products. On the other hand, many theoretical problems in essential are optimal allocation ones in reliability field. The objective of optimal allocation problems is to determine optimal system designs optimizing system reliability [1–7] or other indices [4,8,26] given certain system-level constraints on the system such as cost or weight. Different structures have been developed for this problem such as series [9–11], parallel [11], series-parallel [12–17], k-out-of-n [6,7] and combined series/parallel [2].

Traditionally, the optimal allocation problems were investigated concentrating on reliability systems with single failure mode. In practice, as the system configuration and the failure states of units are becoming more diverse, the modeling of systems with multiple failure states is drawing much attention [18–24]. Diverse failure states will exert different effects on systems and can usually be divided into two classes according to their consequences: dangerous failure state and safe failure state [18,23,24]. In most cases, dangerous failure causes system failure while safe failure simply removes the failed unit from consideration.

For example, Long et al. [25] considered a network of temperature detecting sensors arranged in parallel. A short circuit failure of a single sensor or open circuit failure of all the sensors would make the network unresponsive. In this case, for the system, short circuit failure of a sensor results in system failure and is a dangerous failure state while open circuit failure of a sensor has no impact on the system and is a safe failure state. As a second example, Barlow et al. [24] studied an airline autopilots system containing autopilots arranged in parallel. One of several autopilots may fail by no longer functioning which simply removes the failed autopilot from consideration or by making a destructive maneuver which causes system failure.

In these examples, when a failure unit experiences dangerous failure (safe failure), other units in the same system cannot (can) continue working. In order to model the survival process of units in systems with dangerous failure and safe failure we propose a concept of Sequential Probability Series System (SPSS). SPSS consists of units whose failure states are divided into two classes: dangerous failure and safe failure. Each failure unit independently has probability p of experiencing dangerous failure, $1-p$ of experiencing safe failure. Thus, when a failure unit appears in SPSS, other units in the same system stop working with probability p. Note that when $p=1$, SPSS is the traditional series system; if $p=0$, SPSS is a parallel system; SPSS has more advanced practical

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Table 1
Summary of the proposed three allocation models.

Models	Units	Probability of dangerous failure	Solution methods
Model 1	Identical	A constant	Analytical method & GA
Model 2	Identical	Increases with the number of failure units	GA
Model 3	Non-identical	A constant	Monte Carlo method

applications when $0 < p < 1$. In other words, the proposed SPSS can cover the traditional series and parallel systems.

In addition to industry systems, SPSS exists in many other practical fields such as medical and health (if one patient is infected by an infectious disease in an isolation room, others in the same room will be infected with probability p), storage management (if one item deteriorates in a SPSS, other items in the same SPSS will deteriorate with probability p).

Note that adding units in SPSS reduces the chance of system failure of one type at the same time it will make system failure of the other type more likely. Therefore a practical problem exists in determining the optimal number of units to use in each SPSS. Roughly, optimal allocation problems mainly focus on maximizing system reliability, maximizing expected system life [8] and minimizing cost [26]. From an economic point of view, productivity and profit will be brought by every working unit. In such case, an optimal design of SPSSs maximizing the expected total working time of all units considering certain constraint on budget is more reasonable.

Hence, we can consider the following allocation problem (AP). Allocate a fixed number of units to a certain number of independent SPSSs and find the optimal allocation scheme to maximize the expected total working time of all units. Three allocation models are formulated in this paper which are summarized in Table 1.

Heuristic approaches have been widely applied to handle the optimal allocation problems in reliability such as genetic algorithm (GA) [27–33], particle swarm optimization [34–36], ant colony optimization [37], Immune algorithm [38], variable neighborhood search algorithms [39] and Tabu search (TS) algorithm [40]. In this paper, in cases where analytical solutions are difficult to obtain, an efficient GA and a Monte Carlo method are adopted which can provide us with a reliable optimal result.

The results on optimal allocation policies can be readily applied to many practical systems. We demonstrate the developed three models for SPSS considering a Remote Power Feeding System (RPFS). RPFS is one of the core technologies of the deep seafloor observatory network [41–44] and is playing an increasingly important role in many critical fields. From the configuration and failure mechanism of RPFS, the subsystem of RPFS can be considered as a SPSS. Existing literature mainly focus on reliability analysis of RPFS. Few studies have been conducted that analyze the optimal unit allocation policy for RPFS. The numerical results are presented in each section.

The contribution of this paper is threefold. Firstly, we propose a concept of SPSS which widely exist in many practical systems and consider the optimization of the element allocation maximizing the expected total working time of all units. Secondly, the analytical results on optimal solutions are obtained in certain situations. Lastly, an efficient GA and Monte Carlo method are provided to solve the allocation problem.

The rest of this paper is organized as follows. Section 2 presents an introduction to RPFS and formulates an optimal allocation problem. Section 3 considers the optimal design of RPFS in the case of p being a fixed constant. Firstly, we consider the special case of $p = 1$, and analytical solutions to the problem are provided. Then the case of $0 < p < 1$ is discussed. In this case, the analytical allocation schemes are derived when the lifetime of units obey exponential distributions. A GA is given to search the optimal solutions when the lifetime of units obey general distributions. Section 4 discusses the case of p increasing with the number of failure units. In Section 5, we provide a Monte Carlo method to solve the optimal allocation problem when the lifetime distributions of units are non-identical. Conclusions and some directions of future work are given finally in Section 6. All proofs for theorems presented in the paper are given in the Appendix.

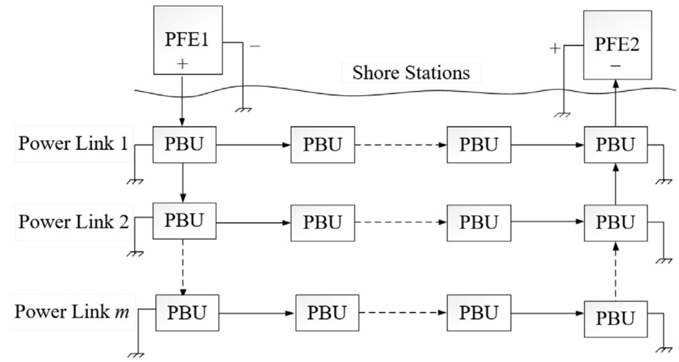

Fig. 1. Engineering model of RPFS.

Acronyms

SPSS	Sequential probability series system
AP	Allocation problem
GA	Genetic algorithm
MC	Monte Carlo
RPFS	Remote Power Feeding System
PBU	Power Branching Unit

Notations

X	Lifetime of a unit
$F_X(x)$	Cumulative distribution function of X
$f_X(x)$	Probability density function of X
$R_X(x)$	Survival function of X, $R(x) = 1 - F(x)$
p	Given a failure has occurred, the conditional probability of dangerous failure
$1 - p$	Given a failure has occurred, the conditional probability of safe failure
n_0	Total number of units to be allocated
α	Maximal number of SPSSs
m	Number of SPSSs
n_i	Number of units in the ith SPSS
$X_{k:n}$	kth smallest order random variable among X_1, X_2, ..., X_n
T_i	Total working time of all units in the $i$thSPSS
$[x]$	Maximal integer not exceeding x
$(n_1, n_2, \cdots, n_m; m)$	Possible optimal allocation scheme if m SPSSs are built
$f(n_1, n_2, ..., n_m; m)$	Expected total working time of all units in all SPSSs if m SPSSs are built
$(n_1^*, n_2^*, \cdots, n_m^*; m)$	Optimal allocation scheme if m SPSSs are built
$(n_1^*, n_2^*, \cdots, n_{m^*}^*; m^*)$	Global optimal allocation scheme

2. Remote Power Feeding System and AP formulation

2.1. Remote Power Feeding System

Remote Power Feeding System (RPFS) is a system in which electrical energy is to be transmitted and is playing more and more critical roles in many practical fields, such as in the field of undersea communication and submarine observation. Fig. 1 shows a typical configuration of RPFS which consists of Power Feeding Equipment (PFE) supplying the constant current, Submarine Cable (SC) providing a conductive path and Power Branching Unit (PBU) keeping current constant.