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Design of validation experiments for life prediction models

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ABSTRACT

This paper proposes a novel validation experiment design optimization (VEDO) method for the assurance of life prediction model, which is one of the key steps in guaranteeing the reliable design of products in meeting the target service life. Life testing data collected from experiments are important for the validation of time-dependent models. However, directly collecting life data for model validation at the operating stress level is usually time-consuming and expensive. In order to overcome this challenge, the accelerated life testing (ALT) method is employed in the proposed method to collect data for model validation. The connection between ALT and model validation is established first; then a VEDO model is developed using the prior information obtained from the computer simulation model. In the VEDO model, the information gain for model validation is maximized within the testing budget and available testing chamber constraints. The obtained optimal number of tests and testing stress levels are designed to maximize the confidence in the validation results. Various sources of uncertainty such as prediction uncertainty, uncertainty of prior information, and observation errors are included within the optimization process in order to improve the robustness of validation experiment design. A composite helicopter rotor hub component is used to demonstrate the effectiveness of the proposed VEDO method.

1. Introduction

The development process of engineering products and systems often includes testing to assess their reliability and service life. In many cases, products and systems may be designed for long service lives and high reliability, and the tests might be expensive for complicated systems. Two types of strategies can be pursued to accelerate the development process and reduce the product development cost, namely accelerated testing and the use of computer simulation models. Accelerated testing subjects the system to elevated levels of demand, thus accelerating its failure; then empirical or assumed relationships are used to estimate reliability or service life under normal operating conditions. Computational models try to emulate the physics of the system and use simulation with assumed distributions of the inputs to predict the product life distribution. However, the life prediction model needs to be validated to make sure it can well represent the actual physical system, before it is applied to the product development. The subject of this paper is effective design of accelerated tests for the validation of life prediction models, thus speeding up the validation process and quantifying the confidence in the estimation of system reliability. Availability of such a quantitative accelerated approach will significantly support product/system development and certification.

Model validation is the process of quantifying the agreement between model prediction and experimental data in order to guarantee that the prediction model can well represent the actual physical system [1]. Model validation can be performed qualitatively (i.e., graphical comparison) or quantitatively (using a validation metric). Many validation metrics have been proposed and investigated during the past decades, such as mean-based methods [2], hypothesis testing-based methods [3,4], area metric [5], and distance or reliability metric [6,7]. In addition to the above metrics, Kullback-Leibler (K-L) divergence is also widely used to measure the discrepancy between two distributions [8]. Based on the quantity of interest and the specific purpose, analysts may select different validation metrics for different problems.

In engineering settings, the validation problems can be either time-independent or time-dependent. Time-dependent problems refer to problems where the output is a function of time, such as fatigue crack growth [9], and stresses and deformation in structures under time-varying loads [10,11]. For time-dependent problems, the service life of products can be predicted from simulation models. For example, a crack growth model can be used to predict the fatigue life of a structure [12] and time-dependent reliability analysis can estimate the time to failure (TTF) of the structure under various sources of uncertainty [13,14]. Validation of time-dependent prediction models (e.g., life prediction model) is usually more challenging than that of time-

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independent models since collection of life data usually requires more effort [15]. Zhang and Mahadevan [15] proposed a Bayesian hypothesis testing-based method to describe the agreement of life interval predictions and observations [15].

In general, two issues have to be considered in the validation testing of life prediction models. The first issue is the financial aspect (testing cost) for validation of products designed for near-infinite service life. The second issue is that the testing conditions/environment be relevant to the operational conditions/environment of the product. The research of this paper is driven by the consideration of the first issue, namely, testing cost. There are two challenges in implementing the method presented in Ref. [15] for the validation of life prediction models belonging to the first issue. First, directly collecting life testing data for validation of time-dependent prediction models of products designed for near-infinite service life is time-consuming and expensive [16]. For problems with very low probability of failure, collecting life testing data under the operational condition will significantly delay the validation process. Second, due to budget constraints, only a limited number of tests may be possible; in that case, confidence in the validation result will depend on the number of tests. Therefore, the data will not only affect the lifecycle cost of product development but also the validation results [17]. In addition, budget and testing facility constraints affect the process of collecting data. In this situation, how to design the experiments to efficiently and effectively collect validation data is an important issue that needs to be addressed.

Experimental design is a process of determining the optimal experimental input settings to maximize the information gain from the experiments. Experimental design methods have been investigated for calibration, validation, or both in the past decades. For example, Huan and Marzouk developed experimental design methods for model calibration using information-theoretic measures and gradient-based stochastic optimization techniques [18,19]; Hu and Mahadevan proposed a calibration experiment design method for time-dependent reliability analysis [20]; Hu et al. investigated the effect of model uncertainty on the results of calibration experiment design [21]; Jiang and Mahadevan proposed a Bayesian cross entropy-based method for validation experiment design of computer simulation models [22]; Jiang and Mahadevan also developed a method to minimize the decision risk, by using likelihood ratio as a validation metric for assessment [17]; and Farrell and Oden applied the calibration and validation experiment design to semiconductor manufacturing [23]. The calibration and validation experiment design methods reviewed above, however, are difficult to apply in the validation of time-dependent prediction models due to the two challenges discussed earlier.

In order to overcome the first challenge (long life), this paper considers the use of accelerated testing, i.e., life testing data collected at stress levels higher than normal stress level, to validate the life prediction model. Here, the stress level refers to any factor that can be used in testing to reduce the time to failure, which can be temperature, force, voltage, humidity, etc [20]. The life testing data are collected at higher stress levels first. Based on that, the life distribution at the normal operational stress level is estimated according to a stress-life relationship. This process is called accelerated life testing (ALT) [20]. By comparing the life distribution obtained from ALT and the prediction model from computer simulation, model validity of life distribution model obtained from the computer simulation is assessed. Based on the proposed ALT-based validation method, the second challenge (variability of test data) is addressed through the formulation of a validation experiment design optimization (VEDO) model, which minimizes the uncertainty in the validation result under the budget and testing chamber constraints. In order to account for various sources of uncertainty present in VEDO, the objective function is formulated in the context of robust design optimization based on pre-posterior variance estimation. Finally, the optimization model is solved using the Fisher information matrix, surrogate modeling, and a genetic algorithm. The obtained optimal testing stress levels and number of tests are expected to maximize the information gain for model validation.

The contributions of this paper are summarized as: (1) formulation of an ALT-based validation approach for time-dependent prediction models in order to reduce the time and cost of model validation; (2) formulation of a novel VEDO model to determine the optimal testing stress levels and number of tests for validation; and (3) integration of ALT statistical model, Bayesian approach, surrogate modeling, and experimental design optimization to solve the proposed VEDO model.

The reminder of paper is organized as follows. In Section 2, background concepts for validation of time-dependent prediction models are briefly introduced. In Section 3, we discuss the proposed VEDO method, which includes ALT-based validation method and VEDO model. A composite rotorcraft hub component example is studied in Section 4 to demonstrate the proposed method. Section 5 provides concluding remarks.

2. Background

2.1. Model validation

In the presence of uncertainty, several approaches have been investigated for quantitative model validation in the literature, and each method has advantages and disadvantages. Current model validation methods can be roughly classified into two groups: namely *hypothesis testing-based methods* and *non-hypothesis testing-based methods* [6].

In hypothesis testing, we decide the plausibility of two hypotheses - the null hypothesis (H_0) and the alternative hypothesis (H_1). The hypothesis testing can be based on classical or Bayesian statistics [6]. For given observation data y_D, in Bayesian hypothesis testing, the Bayes factor [24] given below is used as the validation metric:

$$B = \Pr(y_D|H_0)/\Pr(y_D|H_1) \tag{1}$$

The Bayes factor metric was originally developed to compare the data support for two physics models. It has been extended to compare two competing probability distribution models with uncertain parameters as follows [4]:

$$B = \Pr(y_D|H_0)/\Pr(y_D|H_1) = \int \Pr(y_D|\boldsymbol{\theta}_i)\pi(\boldsymbol{\theta}_i)d\boldsymbol{\theta}_i / \int \Pr(y_D|\boldsymbol{\theta}_j)\pi(\boldsymbol{\theta}_j)d\boldsymbol{\theta}_j \tag{2}$$

Bayesian hypothesis testing has also been investigated for equality hypotheses [3], interval hypotheses [3], and for validation data from fully or partially characterized experiments [25].

In non-hypothesis testing-based methods, the commonly studied validation metrics include the Mahalanobis distance [26], K-L divergence [27], area metric-based methods [5,28], and reliability-based metric [6,7]. Here, we briefly review the area metric and reliability-based metric since they have clear physical or probabilistic interpretations in terms of model validity, and both can be applied to validation of a model with multiple input variables using data from discrete test combinations [6].

2.1.1. Area metric

The area metric proposed by Ferson er al. [1,6] quantifies the mismatch between prediction and observation data using the area between the cumulative distribution (CDF) of model prediction and experimental data. This area can be expressed as

$$d(F_{ym}, S_{yn}) = \int_{-\infty}^{+\infty} |F_{ym}(y) - S_{yn}|dy \tag{3}$$

where F_{ym} represents the CDF of model prediction and S_{yn} is the empirical CDF of experimental data.

The area metric can also be transformed from the physical space to probability space using the “u-pooling” procedure [29,30]. After the