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A confidence-based approach to reliability design considering correlated failures

Lance Fiondella[a,1], Yi-Kuei Lin[b,*], Hoang Pham[c], Ping-Chen Chang[d], Chendong Li[e]

[a] *Department of Electrical and Computer Engineering, University of Massachusetts, 285 Old Westport Road, Dartmouth, MA 02747, USA*
[b] *Department of Industrial Management, National Taiwan University of Science & Technology, Taipei 106, Taiwan ROC*
[c] *Department of Industrial Engineering, Rutgers University, Piscataway, NJ 08854, USA*
[d] *Department of Industrial Engineering and Management, National Quemoy University, Kinmen County 892, Taiwan ROC*
[e] *Dell Inc., 5455 Great America Parkway, Santa Clara, CA 95054, USA*

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ABSTRACT

To maintain a competitive edge, technology manufacturers must produce systems that are reliable enough to satisfy customers yet cheap enough to engineer so that they are profitable. This paper presents an optimization model to maximize the statistical confidence in product profitability, permitting flexibility in the design and number of the units manufactured. This is unlike traditional approaches, which focus on the two cases that optimize the reliability of a single unit or the *s*-expected profit obtained from a very large number of units. These two extremes disregard a practical concern, namely the negative impact that a larger than *s*-expected number of failures will exert on product profitability. This paper formulates an optimization problem to mitigate this risk. Virtually all reliability optimization problems also assume that component failures are *s*-independent. The present paper does not impose this assumption. The utility of the approach is demonstrated through a series of examples which compare the reliability of systems designed with and without the assumption of *s*-correlated component failures. The results indicate that explicitly considering *s*-correlation consistently mitigates the risk to profitability more effectively than the same method when component failures are assumed to be *s*-independent.

1. Introduction

Reliability and cost are competing constraints in manufactured systems.[2] Reliability is essential to achieve a desired level of customer satisfaction. On the other hand, cost control is critical to maintain product profitability. High reliability alone will not guarantee product viability because production cost must be managed. Similarly, arbitrary cost cutting can be detrimental to profit when the resulting system reliability is too low. Thus, a compromise between these two factors is necessary to optimize profitability. While the reliability of each unit and the overall profitability are desirable attributes of any system design, a methodology to address the uncertainty inherent in the production of a finite number of units is needed to mitigate risk.

Two approaches to optimize the reliability of commercial off-the-shelf systems dominate the research literature. The first [1] makes critical components fault-tolerant to improve system reliability. One shortcoming of the redundancy allocation approach is that virtually all systems are produced in quantities greater than one and there is no guarantee that what is cost optimal for a single unit will be cost optimal for a larger number of items. Furthermore, the vast majority of reliability optimization techniques rely on the assumption that component failures are statistically independent, which can lead to optimistic overestimates of system reliability. A second popular approach to optimal reliability design [2] attempts to maximize the *s*-expected profit of a unit from an arbitrarily large population. A limitation of the profit maximizing approach is that not all products are capable of achieving an *s*-expected value because variance in the number of failures from a finite lot can introduce non-trivial variance into the actual profit derived. Recent contributions cost-informed reliability optimization include the work of Amari et al. [3] who proposed optimal cost-effective design policies for *k*-out-of-*n*:G subsystems that can experience imperfect fault-coverage and Amari and Pham [4], which

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* Corresponding author.
E-mail addresses: lfiondella@umassd.edu (L. Fiondella), yklin@mail.ntust.edu.tw (Y.-K. Lin), hopham@rci.rutgers.edu (H. Pham), pcchang@nqu.edu.tw (P.-C. Chang), chendong.lcd@gmail.com (C. Li).

[1] Tel.: +1(508)999-8596.

[2] The terms system and unit are used interchangeably in this paper.

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Nomenclature

x_i	Component redundancy in position i.
p	Number of positions in system architecture.
ϕ	System structure function.
$E[R_s]$	s-expected system reliability.
m	Number of resource constraints.
b_i	Budget constraint of ith resource.
g_i	Resource i consumption function.
$g_{i,j}$	Resource i consumed by redundancy in position j.
l_j	Redundancy lower bound of position j.
u_j	Redundancy upper bound of position j.
C_r	Reward for a reliable system.
C_m	Cost to manufacture a system.
C_l	Loss from an unreliable system.
F	Funds available for manufacturing.
N	Number of systems manufactured.
P	Random variable for profit of system design.
$E[P]$	s-expected profit of system design.
P^*	Target profit of system design.
α	Probability P^* is not achieved.
μ	s-expected component reliability.
n	Number of redundant components.
μ'	Reliability of parallel system composed of n components of reliability μ.
ρ	Correlation between component failures.
μ''	Reliability of parallel system composed of n components of reliability μ and correlation ρ.
$\boldsymbol{\mu}$	$1 \times n$ vector of non-identical s-expected component reliabilities.
μ_i	s-expected reliability of component i.
$\boldsymbol{\Sigma}$	$n \times n$ correlation matrix.
$\rho_{i,j}$	Correlation between failures of components i and j.
$\mathbf{R}$	$1 \times n$ vector of component states.
r_i	State of component i, $r_i \in \{0, 1\}$.
$\lambda_{i,j}$	Poisson rate parameter encoding correlation between components i and j.
$\boldsymbol{\Lambda}$	$n \times n$ upper diagonal matrix of rate parameters.
$\boldsymbol{\Lambda}^k$	$\boldsymbol{\Lambda}$ matrix in iteration k.
$\lambda_{i,j}^k$	$(i, j)^{th}$ entry of $\boldsymbol{\Lambda}^k$.
$\lambda_{\min}^k$	Minimum $\lambda_{i,j}^k > 0$.
$\boldsymbol{\lambda}_{\min}$	$1 \times n(n+1)/2$ vector of $\lambda_{\min}^k$.
X_k	kth Poisson variable, with rate $\lambda_{\min}^k$.
$\mathbf{X}$	$1 \times n(n+1)/2$ Poisson vector encoding component correlations.
S^k	Set of components to which $X_k(\lambda_{\min}^k)$ is added.
$\mathbf{S}$	$1 \times n(n+1)/2$ vector of S^k.
$S^{i,j}$	Set of components that fail when $X_i = 1$ and $X_j = 1$.
$I_{\{0\}}(\psi)$	Indicator function. $I_{\{0\}}(\psi) = 1$ for $\psi = 0$.
C	Cutsets of system.
c	Individual cuts $c \in C$.
$\boldsymbol{\Theta}$	$1 \times n(n+1)/2$ vector of Poisson outcomes.
$C(\boldsymbol{\Theta})$	Set of component failures given $\boldsymbol{\Theta}$.
$z_{i,j}$	jth node at depth i of branch and bound tree.
Z_i	Set of nodes at depth i.

minimizes the cost of complex repairable systems by identifying the optimal number of spares for each subsystem. However, these recent contributions continue to assume that the s-expected profit can be attained by producing a large quantity of items. Thus, an approach that quantifies the impact of s-correlated component failures on the reliability of a system and a larger than s-expected number of failures in a small lot of items will complement existing techniques well.

Two methods are frequently used for modeling the choice among uncertain outcomes: stochastic dominance and mean-risk approaches [5]. Value at Risk (VaR) measures an investment's risk by estimating how much an investment might lose given normal market conditions in a specified period of time and it is used by firms and regulators [6] in the financial industry to assess the assets that may be needed to cover possible losses. Conditional Value at Risk (CVaR) [7] is the expected return on a portfolio in the worst cases, which is intended to be more sensitive to the shape of the tail of the loss distribution. This paper presents an approach to manage uncertainty when a system is produced in smaller quantities. Instead of maximizing the s-expected profit, the approach identifies a design to maximize the statistical confidence that a desired profit will be realized. The goal is to identify the ideal combination of a reliable system design and the quantity of units to be produced given a limited budget. Thus, it becomes possible to manufacture a small number of highly reliable units of high quality or a larger number of lower quality units with lower reliability. This approach allows an organization to study the potential risk and reward of alternative designs. In addition to the greater realism enabled by considering the production of a finite number of units, the proposed approach also removes the widespread and unrealistic assumption that redundant components fail in a statistically independent manner.

Algebraic expressions [8] and numerical algorithms [9] to quantify the impact of s-correlation on discrete system reliability are utilized. The algebraic expressions are applicable to several common structures, including series, parallel, and series-parallel systems, are computationally efficient, and therefore suitable for intensive calculations performed during optimization. The numerical algorithms are applicable to the broader class of coherent systems [1] and can therefore also consider systems possessing complex network structures. Our previous papers [8–10] were restricted to the modeling and sensitivity analysis of s-correlation on system reliability and have not been applied in the context of any reliability optimization problem. We demonstrate the effectiveness of these approaches for solving the s-confidence optimization problem through a series of examples. The results indicate that the proposed techniques integrating the assumption of s-correlated component failures outperform the same techniques when component failures are assumed to be s-independent, thereby mitigating the negative impact of correlation on reliability while simultaneously maximizing the s-confidence profitability exceeds a desired target. Thus, the proposed approach can provide useful insight to managers, engineers, and scientists wishing to understand how correlated failures in the components of a system may impact its reliability and the potentially negative influence a larger than s-expected number of failures will exert on product profitability.

The paper is organized as follows: Section 2 summarizes related research. Section 3 outlines two widely accepted optimization models. Section 4 discusses some limitations of these previous approaches and proposes techniques to remedy these shortcomings. Section 5 provides illustrations. Section 6 offers conclusions and future research.

2. Related research

This section reviews the related research along multiple dimensions, including reliability optimization problems, the redundancy allocation problem, optimizing average system cost, correlated or dependent component failure times, and uncertainty and risk aversion for system reliability optimization.

2.1. Reliability optimization

Many techniques have been applied to reliability optimization problems including heuristic methods, dynamic programming [11],