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# Maintenance optimization under non-constant probabilities of imperfect inspections

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ABSTRACT

In this research, we study a single-component system that is characterized by three distinct deterioration states, cf. the Delay Time Model: normal, defective, and failed. The system is inspected periodically, and preventive system maintenance is done after a given number of inspections. The inspections are imperfect, and the probability of an inspection error changes over the system's operation time. Our objective is to minimize the average cost over an infinite time horizon. We present exact cost evaluations for a given maintenance policy, and we compare our model with non-constant probabilities to a model that considers constant probabilities of inspection errors. Our computational study illustrates that the model with constant probabilities may yield, on average, 19% higher costs than the model using non-constant probabilities of inspection errors. These values depend on the chosen parameter values, but still give an indication of how large the difference between both models can be. Finally, we also present an extension in which a reliability constraint (in terms of average failures per time unit) is added to our problem.

## 1. Introduction

Unexpected failures cause costly downtime for many advanced technical systems, such as airplanes, trains, baggage handling systems, and medical systems. Maintenance is done during system operation to avoid such unexpected failures. The costs of these maintenance activities comprise 15–60% of the total production costs in a manufacturer's facility [11]. In such situations, it is important to minimize the maintenance costs. Mathematical maintenance models and techniques are typically used to support this objective by deriving optimal maintenance policies.

The literature on maintenance optimization is rich and covers various areas such as system replacement, inspections, repair, and maintenance scheduling [7]. These areas of maintenance optimization use modeling techniques that describe system degradation. There exists a large variety of models describing system degradation differently, e.g. continuous degradation or discrete state degradation. Many of these models have in common that they also consider inspections that reveal the system's level of degradation. For instance, Whitmore [22], and Newby and Dagg [12] model the continuous degradation by a Wiener process and assume imperfect inspections that may contain noise. In other work, Kallen and van Noortwijk [9] propose to model system degradation by a Gamma process, and they also assume inspections with noise. On the other hand, system degradation is also modeled in terms of discrete states, contrary to continuous degradation. Some researchers have proposed multi-state Markovian degradation models that include imperfect inspections, see for example Welte [21], and Le and Tan [10].

Another common approach to modeling system degradation in terms of discrete states is the Delay Time Model (DTM). This model distinguishes three deterioration states: normal, defective, and failed. The system operates properly in the normal state; operates in the defective state as well, but its defect can be revealed by inspections; or the system has failed. The time the system spends in the normal and defective state are called the time to defect and the delay time, respectively. Analogous to other maintenance models in literature, the DTM is typically studied under inspection based maintenance policies; i.e., inspections are done to reveal the system's degradation. A literature overview of the DTM up to 1999 is provided by Christer [4], and Wang [19] reviews the research from 1999 to 2012. The most recent advancements, since 2012, include postponements of maintenance actions when the defects are detected [17], and the combination of multiple different forms of preventive maintenance activities – such as routine service, preventive system maintenance, and inspections – based on the DTM [20].

The DTM literature considering inspection policies generally as-

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**Table 1**
Probabilities of inspection behavior.

| | | System state | |
|---|---|---|---|
| | | Normal | Defective |
| Inspection outcome | Normal | $1-\alpha$ | $\beta$ |
| | Defective | $\alpha$ | $1-\beta$ |

sumes that the inspections are perfect. However, inspections are usually not perfect in practice due to errors, such as human errors or errors of measurement equipment [23]. Therefore, imperfect inspections have been included in single-component DTMs by Okumura et al. [14] and Berrade et al. [3]. Both works consider two types of imperfect inspection behavior, i.e., false positives and false negatives. A false positive corresponds to the judgment that a system is in its defective state, when it is actually in its normal state. False negatives correspond to the judgment that a system is in its normal state when, in fact, the system is in its defective state. We refer to the probabilities of false positives and false negatives by $\alpha$ and $\beta$, respectively. For an overview, Table 1 is included.

The probabilities of false positives and false negatives, $\alpha$ and $\beta$, are assumed to be constant over time by Okumura et al. [14] and Berrade et al. [3]. However, this approach might be inaccurate. Maintenance engineers may become more tempted to engage in a false positive when the system has been operating (in its normal state) for a longer period of time. That is, they use their own judgement of the time the system has been operating normally instead of the actual inspection outcome. Such behavior implies that the probability of false positives is non-constant, and depends on the time the system has been operating in the normal state. This approach to false positives has not been explored in the DTM literature, nor in any of the other maintenance models. Therefore, our work extends the literature by proposing a non-constant probability of false positives for a single-component DTM. Furthermore, maintenance engineers may encounter difficulties in determining whether a system has become defective at early stages of system degradation, e.g. smaller cracks are more difficult to find than larger ones [5]. Therefore, the probability of false negatives is non-constant and depends on the system degradation. We extend the literature by relating this probability of false negatives to the system's duration in the defective state relative to its delay time, which is a measure of system degradation. By this way of modeling the probability of false negatives, we provide a richer concept of system degradation compared to Wang [18], who conceptualizes system degradation by only considering the duration that a system has been defective. Let us illustrate the enhanced richness of our concept by means of an example. Weaker materials typically have a lower delay time compared to stronger materials. Hence, under the same duration in the defective state, the weaker material (shorter delay time) will have a higher level of degradation than the strong one. Table 2 presents a schematic literature overview of DTMs with imperfect inspections.

In this research, we propose a single-component model that considers non-constant probabilities of false positives and false negatives, and we will refer to this model as the *true model*. Next to the true model, we also study the *approximate model*, which is a single-component model that considers constant probabilities of false positives and false negatives. This approximate model is typically easier to use and to implement due to the constant probabilities. Furthermore, the approximate model will serve as a comparison to the true model in this paper. This enables us to shed some light on the benefit of modeling non-constant probabilities of imperfect inspections over modeling the probabilities as constants. Our objective is to minimize the average cost per time unit over an infinite time horizon by optimizing the maintenance policy. Our research's contributions are twofold: (1) we present an exact cost evaluation of our true model; and (2) we compare our true model to the approximate model. We show that the approximate model may result in policies that yield – on average – 19% higher cost than the true model. This implies that the reduction in model complexity comes at an average cost increase of 19%, based on our instances tested. We would like to emphasize that these numbers depend on the parameter settings, but they still give an indication of the extent that the two models may differ.

**Table 2**
Imperfect inspections in the DTM literature.

| | | False positives | |
|---|---|---|---|
| | | Constant | Non-constant |
| False negatives | Constant | Okumura et al. [14]; Berrade et al. [3] | |
| | Non-constant | Wang [18] | This research |

The remainder of this paper is organized as follows. In Section 2, we present the model. In Section 3, we give an exact cost evaluation for our maintenance policy, and we discuss the optimization procedure. In Section 4, we present a method for comparing the true and approximate model, and we present the computational results that compare both models in Section 5. We present an extension including a reliability constraint in Section 6, and we conclude our work in Section 7.

## 2. Model description

In this section, we describe our true model. However, the description and the reasoning also applies to the approximate model. The sole difference is that, in the latter case, the probabilities of false positives and false negatives are assumed to be constants.

Let us consider a single-component system operating over an infinite time horizon. The system has three states: normal, defective and failed. In the normal operating state, the system is working properly, without any detectable defects. In the defective state, inspections may reveal the system's defect. Yet, the system is still able to operate. The failed state of the system is self-announcing, and the system stops delivering its function immediately. If the system fails, it is replaced correctively. To prevent the system from reaching its failed state, it is inspected periodically each $T > 0$ time units, it is preventively replaced upon detection of the defective state, or it is preventively replaced after $M - 1$ inspections at time $MT$. This implies that at time $MT$ we do not perform another inspection, as the system is preventively replaced independent on its state, i.e., the $M^{th}$ inspection is unnecessary. For more details on such a policy, see Scarf et al. [16]. We denote our maintenance policy by $(M,T)$, and note that it degenerates to an age-based maintenance policy when $M$=1, it reduces to a pure inspection policy when $M = \infty$, and it is a hybrid policy for any finite $M > 1$. We assume that inspections are the only means to detect the normal and defective state.

We denote the duration of the time that the system is in the normal state, referred to as the time to defect, by the continuous random variable $X > 0$. This time to defect corresponds to the time between maintenance (preventive or corrective), and the arrival time of the defect. The random time the system takes from defect arrival to failure, without doing any maintenance, is referred to as the delay time and denoted by the continuous random variable $H > 0$. The sum of both random variables is the system's time to failure. The cumulative distribution function (cdf) and the probability density function (pdf), corresponding to both state durations, are defined by $F_X(\cdot)$ and $f_X(\cdot)$ for the time to defect, and by $F_H(\cdot)$ and $f_H(\cdot)$ for the delay time, respectively.

The cost for performing an inspection is denoted by $c_i$, and the cost