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Multilevel Monte Carlo for Reliability Theory

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ABSTRACT

As the size of engineered systems grows, problems in reliability theory can become computationally challenging, often due to the combinatorial growth in the number of cut sets. In this paper we demonstrate how Multilevel Monte Carlo (MLMC) — a simulation approach which is typically used for stochastic differential equation models — can be applied in reliability problems by carefully controlling the bias-variance tradeoff in approximating large system behaviour. In this first exposition of MLMC methods in reliability problems we address the canonical problem of estimating the expectation of a functional of system lifetime for non-repairable and repairable components, demonstrating the computational advantages compared to classical Monte Carlo methods. The difference in computational complexity can be orders of magnitude for very large or complicated system structures, or where the desired precision is lower.

1. Introduction

It can prove to be computationally intractable to perform classical reliability analysis of very large engineered systems when the number of cut (path) sets grows combinatorially. It is well understood that working instead with subsets of the cut (path) sets or bounding structural designs can provide probability bounds in many reliability problems [4], but such bounds can be crude or may not be well characterised at all.

Evaluation of the reliability of engineered systems is a crucial part of system design and often scenario planning may involve repeated evaluation of the reliability for changing system configurations or component types meaning rapid simulation is highly desirable. For simplicity of exposition we herein consider the canonical problem of estimating the expectation of a functional of system lifetime both with and without a component repair process, showing the approach developed is easily generalised to other reliability problems which depend on cut (path) sets for the analysis.

In the case of static reliability analysis, there are many methods aside from Monte Carlo simulation using the cut (path) sets, including Sum of Disjoint Products (SDP) methods [22,30,27] and methods based on Binary Decision Diagrams (BDD) [25] or multistate BDD extensions [29]. On the other hand, these approaches are less prevalent in dynamic reliability problems where there are component dependencies, for example through system shocks, repair or maintenance programmes, and cascading failures among others. There have been recent developments in dynamic fault trees [20,26,21] which apply where event sequence ordering influences the reliability, including repairable systems [19]. When there are arbitrary dependencies, the most generally applicable approach is direct Monte Carlo simulation (e.g. [7]), so that acceleration of Monte Carlo techniques is important to address a broad range of the most complex reliability scenarios. Monte Carlo acceleration through importance sampling [15], or the use of control variates [28] have been suggested in the context of reliability estimation, but they are either restricted to the static case and require regular updates and sorting of all the cut sets (as for [15]), or could be combined with the MLMC paradigm (as for [28]).

Indeed, also note that interest may not be in the reliability at a particular fixed mission time, but instead in: some expectation of a functional of system lifetime; or in ascertaining a quantile of system lifetime (i.e. the time to which one is 99.9% certain the system will survive); or in estimation of the entire system lifetime distribution. In these situations Monte Carlo methods are typically the only tractable approach.

Multilevel Monte Carlo (MLMC) methods — pioneered by Heinrich [14] and Giles [10] — are now standard for estimation of expectations of functionals of processes defined by stochastic differential equations (SDEs). However, the MLMC approach is in fact a general paradigm for accelerating any Monte Carlo based method (whether standard, importance sampling, etc), if one can link the accuracy of the estimator with the complexity of generating a sample, while at the same time controlling the variance of the difference for approximations with different accuracy. The main contribution in this paper is development of a Multilevel Monte Carlo (MLMC) approach to

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reliability problems. In this way, we show how any reliability problem using Monte Carlo simulation over cut (path) sets can be substantially accelerated, extending the size of systems and complexity of dependencies which are within reach for reliability evaluation.

In Section 2, we recap the traditional cut set method of simulating system lifetimes which does not scale well to large systems even when the cut sets are known. This motivates the approach taken in this work. In Section 3 we recap standard Monte Carlo theory and set out the error and computational cost metrics which will enable comparison with our MLMC based approach. The fundamental MLMC methodology and our adaptation to the reliability setting then follow in Section 4, before numerical results demonstrating the kind of substantial computational improvements which can be achieved are covered in Section 5,6.

2. Simulating system lifetimes

Consider a coherent system with n components. Let $x_1(t), \ldots, x_n(t)$ denote the operational status (1= working, 0= failed) of the components at time t and consider the random variable for the lifetime of component c to be $T_c \sim F_c(t)$, where $F_c(\cdot)$ are positively supported lifetime distributions which are not necessarily independent or identical. We will depict a system as an undirected network comprising a set of nodes (vertices) S, and a set of edges E, where nodes are considered unreliable and edges are perfectly reliable (note that any setting with imperfect edge reliability can be easily transformed to a corresponding representation where they are perfectly reliable [2]).

The system is considered to be functional as long as there is a path from left to right which passes only though functioning nodes, see Fig. 1. This is usually represented mathematically by the structure function, ϕ: $\{0, 1\}^n \to \{0, 1\}$, which maps component status to system status.

Herein, our focus is on an equivalent means of evaluation based on cut sets. A set of components, C, is said to be a cut set of the system if the system is failed whenever all the components in C are failed. A cut set is said to be a minimal cut set if no subset of it is also a cut set. Then, the set of all minimal cut sets, C, characterises the operational state of a system completely and is equivalent to knowledge of the structure function [6]. In addition to the cut sets characterising the operational state of the system given the binary operational state of the components, they also immediately provide the system failure time if the individual component failure times are known [5]:

$$T_S = f_S(T_1, \ldots, T_n) := \min_{C \in C} \left\{ \max_{c \in C} \{T_c\} \right\}.$$

Thus, the failure time for the system depends on the system structure and the failure time distributions for each node.

The traditional approach to estimating the expectation of a functional of the lifetime of a system given the lifetime distributions of the components is to perform a simple Monte Carlo simulation. That is,

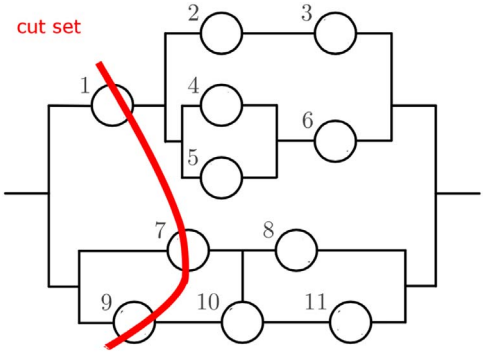

Fig. 1. A sample network with a minimal cut set.

$$\mathbb{E}[g(T_S)] = \frac{1}{N} \sum_{i=1}^{N} g(f_S(t_1^{(i)}, \ldots, t_n^{(i)})) \text{ where } t_j^{(i)} \sim F_j(\cdot).$$

The overall runtime for this approach depends on three quantities:

1. Variance of the estimator. Due to the random nature of component failure times, the estimator is a random variable: higher variance estimators will require more iterations to achieve an accurate estimate;
2. Target accuracy of the estimate. Naturally, the higher the desired accuracy, the longer the algorithm will take due to more iterations being required;
3. Number of cut sets. As the system size grows, the number of cut sets has a combinatorial growth, making the approach impractical for very large systems.

Less brute force approaches are possible with the restrictive assumption of iid components by making use of the system signature [18,23]. More recent work on the survival signature [8] has generalised the signature to multiple types of component, with the weaker assumption of exchangeability between components. However, if a large number of the components are of different types or there are highly dependent failures, then the survival signature will also grow exponentially in complexity. It can also accommodate a repair process [9] through expression as a new component type, though this increases the complexity if too many repairs occur. Hence, in this work, we first address the most general possible setting in which any form of component lifetime and dependence structure is allowed, requiring only knowledge of component lifetimes and the cut sets of the system. However, note that it should be possible to specialise this approach to work with the survival signature which we hope to address in future research.

3. Monte Carlo algorithms

To simplify presentation, hereinafter we only consider estimating expected failure time directly, rather than some functional of the failure time, though this is mostly without loss of generality (see Section 4 for details). Therefore, assume that for a given system S, we want to estimate the expected failure time.

$$\mathbb{E}T_S = \mathbb{E}f_S(T_1, \ldots, T_n).$$

There are many approaches to simulation which may differ in terms of convergence to the true value as well as computational characteristics. In order to compare them, we present some useful cost and error expressions in the following subsection.

3.1. Performance measures: error and cost definitions

We start by defining the two main quantities, which will be used throughout this paper to compare methodologies. Given an estimator $\hat{T}_S$ of the quantity $\mathbb{E}T_S$, the Mean Squared Error (MSE) of any Monte Carlo based method is:

$$\text{error} = \mathbb{E}\left[\left(\hat{T}_S - \mathbb{E}T_S\right)^2\right].$$

The classical decomposition of the MSE yields:

$$\mathbb{E}\left[\left(\hat{T}_S - \mathbb{E}T_S\right)^2\right] = \mathbb{E}\left[\left(\hat{T}_S + \mathbb{E}\hat{T}_S - \mathbb{E}\hat{T}_S - \mathbb{E}T_S\right)^2\right] = \mathbb{E}\left[\left(\hat{T}_S - \mathbb{E}\hat{T}_S\right)^2\right] + \left(\mathbb{E}\hat{T}_S - \mathbb{E}T_S\right)^2 \quad (1)$$

where $\left(\mathbb{E}\hat{T}_S - \mathbb{E}T_S\right)^2$ is the squared bias error, while $\mathbb{E}[(\hat{T}_S - \mathbb{E}\hat{T}_S)^2]$ is the error due to Monte Carlo variance. The first is a systematic error arising from the fact that we might not sample our random variable exactly, but rather use a suitable approximation, while the second error comes from the randomised nature of any Monte Carlo algorithm. For example, in traditional Monte Carlo applications, one samples exactly

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