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An efficient algorithm for exact computation of system and survival signatures using binary decision diagrams

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ABSTRACT

System and survival signatures are important and popular tools for studying and analysing the reliability of systems. However, it is difficult to compute these signatures for systems with complex reliability structure functions and large numbers of components. This paper presents a new algorithm that is able to compute exact signatures for systems that are far more complex than is feasible using existing approaches. This is based on the use of reduced order binary decision diagrams (ROBDDs), multidimensional arrays and the dynamic programming paradigm. Results comparing the computational efficiency of deriving signatures for some example systems (including complex benchmark systems from the literature) using the new algorithm and a comparison enumerative algorithm are presented and demonstrate a significant reduction in computation time and improvement in scalability with increasing system complexity.

1. Introduction

The system signature, introduced by Samaniego [1], is a useful tool for studying the reliability of coherent systems [2]. Consider a coherent system of m components with independent identically distributed failure times. Let $T_s>0$ be the random failure time of the system and let $T_{j:m}$ be the *lth* order statistic for the random component failure times with $T_{1:m}\le T_{2:m}\le\ldots\le T_{m:m}$. The system signature is defined as the vector q where the value at index $l\in\{1,2,\ldots,m\}$, denoted q_l, gives the probability that the system failure time coincides with the *lth* component failure

$$q_l = P(T_s = T_{l:m}) \tag{1}$$

The system signature has various theoretical applications in reliability engineering such as establishing stochastic comparisons between the reliability of different systems [3,4]. An overview of the system signature and some of its applications in reliability engineering is given by Samaniego [2], whilst Eryilmaz [5] gives a review of recent advances. Recently, Coolen and Coolen-Maturi [6] introduced the survival signature which, similar to the system signature, fulfils the role of a quantitative model of the system reliability structure that is entirely separated from the random failure times of the components. The survival signature has the advantage that is can be easily generalised to systems with multiple types of components unlike the system signature for which this is practically impossible [6]. This generalisation represents a significant practical advantage since many systems contain multiple component types, including networks which contain at least two types of component ('nodes' and 'links'). Let $x=(x_1,x_2,\ldots,x_m)\in\{0,1\}^m$ represent a Boolean state vector for a system of m components with exchangeable failure times, where x_i=1 if component i functions and x_i=0 if it is failed. Also let $\phi:\{0,1\}^m\to\{0,1\}$ represent the system reliability structure function, defined for all 2^m possible x, where $\phi(x)$=1 if the system functions with component states x and $\phi(x)$=0 if it is failed. Finally, let S_l denote the set of component state vectors with exactly$l$ of the m components functioning (i.e. $\sum_{i=1}^{m}x_i=l$). The survival signature is then defined as the vector Φ where the value at index $l\in\{0,1,2,\ldots,m\}$, denoted Φ_l, gives the probability that the system functions given that precisely/components function

$$\Phi_l = \binom{m}{l}^{-1}\sum_{x\in S_l}\phi(x) \tag{2}$$

Now consider the case where the m components in the system are partitioned into K different types, where the M_k components of type $k\in\{1,\ldots,K\}$ have exchangeable random failure times. Let $S_{l_1,\ldots,l_K}$ denote the set of component state vectors that contain precisely $l_k\in\{0,1,\ldots,M_k\}$ functioning components of type k (i.e. those for which $\sum_{i=1}^{M_k}x_i^k = l_k$ for $k=0,1,\ldots,K-1$ where x_i^k is the *ith* component of type k). Also let $|S_{l_1,\ldots,l_K}| = \prod_{k=1}^{K}\binom{M_k}{l_k}$ denote the cardinality of $S_{l_1,\ldots,l_K}$ and $\overline{\Phi}_{l_1,\ldots,l_K} = \sum_{x\in S_{l_0,\ldots,l_{K-1}}}\phi(x)$ denote the number of state vectors from $S_{l_1,\ldots,l_K}$ for which the system functions. The generalised survival signature, Φ, is then defined as the multidimensional array with K

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Nomenclature

m	Number of components in the system
K	Number of component types in the system, where components of the same type have exchangeable random failure times
M_k	Number of components of type k
x_i	Boolean variable representing the state of component i where x_i=1 if the component functions and x_i=0 if the component is failed
x	Vector of length m representing the system component states where the value at index $i \in \{1,\ldots,m\}$ corresponds to x_i
ϕ	Boolean function representing the system reliability structure where $\phi(x)$=1 if the system functions with component state vector x and $\phi(x)$=0 if the system is failed
$f_{x_i=v}$	Boolean function f evaluated with Boolean variable x_i=v
T_s	Random failure time of the system
$T_{l:m}$	lth order statistic for the random component failure times
q_l	Probability that the system failure time coincides with the lth component failure (i.e. $P(T_s = T_{l:m})$)
q	Vector of length m known as the system signature where the value at index $l \in \{1,\ldots,m\}$ corresponds to q_j
$S_{l_1,\ldots,l_K}$	Set of state vectors for the m components that contain precisely l_k functioning components of type k
$\lvert S_{l_1,\ldots,l_K}\rvert$	Cardinality of $S_{l_1,\ldots,l_K}$
S	Multidimensional array with K dimensions where the value at index $(l_1,\ldots,l_K)$ in dimensions $(1,\ldots,K)$ respectively corresponds to $S_{l_1,\ldots,l_K}$
$\lvert S\rvert$	Multidimensional array with K dimensions where the value at index $(l_1,\ldots,l_K)$ in dimensions $(1,\ldots,K)$ respectively corresponds to $\lvert S_{l_1,\ldots,l_K}\rvert$
$\overline{\Phi}_{l_1,\ldots,l_K}$	Number of state vectors for the m components that both contain precisely l_k functioning components of type k and result in the system functioning
$\overline{\Phi}$	Multidimensional array with K dimensions where the value at index $(l_1,\ldots,l_K)$ in dimensions $(1,\ldots,K)$ respectively corresponds to $\overline{\Phi}_{l_1,\ldots,l_K}$
$\Phi_{l_1,\ldots,l_K}$	Probability that the system functions given that exactly $(l_1,\ldots,l_K)$ components of types $(1,\ldots,K)$ respectively function
Φ	Multidimensional array with K dimensions known as the survival signature where the value at index $(l_1,\ldots,l_K)$ in dimensions $(1,\ldots,K)$ respectively corresponds to $\Phi_{l_1,\ldots,l_K}$
C_t^k	Number of components of type k in the system that function at time $t > 0$
F_k	Cumulative distribution function for the time to failure of components of type k
$A^{\ddagger}$	The complement of multidimensional array A
$A \oplus B$	Elementwise addition of multidimensional arrays A and B
$A \ominus B$	Elementwise subtraction of multidimensional array A from multidimensional array B
$A \oslash B$	Elementwise division of multidimensional array A from multidimensional array B
$A \boxplus k$	Resize-k operation on multidimensional array A
$A \circledast k$	Shift-k operation on multidimensional array A

dimensions where the value at index $(l_1 \in \{0,..,M_1\},\ldots,l_K \in \{0,..,M_K\})$ in dimensions $(1,\ldots,K)$ respectively, denoted $\Phi_{l_1,\ldots,l_K}$, gives the probability that the system functions given that precisely $(l_1,\ldots,l_K)$ components of types $(1,\ldots,K)$ respectively function

$$\Phi_{l_1,\ldots,l_K} = \frac{\overline{\Phi}_{l_1,\ldots,l_K}}{\lvert S_{l_1,\ldots,l_K}\rvert} \tag{3}$$

Let $C_t^k \in \{0,\ldots,M_k\}$ denote the number of components of type k in the system that function at time $t > 0$. The probability that the system functions at time t can be calculated using the survival signature and the joint probability distribution for the number of functioning components of each type at time t

$$P(T_S > t) = \sum_{l_1=0}^{M_1}\ldots\sum_{l_K=0}^{M_K}\left[\Phi_{l_1,\ldots,l_K} P\left(\bigcap_{k=1}^{K}\{C_t^k = l_k\}\right)\right] \tag{4}$$

If failure times of components of type k are conditionally independent and identically distributed with CDF $F_k(t)$ and failure times of components of different types are independent, then

$$P(T_S > t) = \sum_{l_1=0}^{M_1}\ldots\sum_{l_K=0}^{M_K}\left[\Phi_{l_1,\ldots,l_K}\prod_{k=1}^{K}\left(\binom{M_k}{l_k}[F_k(t)]^{M_k-l_k}[1-F_k(t)]^{l_k}\right)\right] \tag{5}$$

For systems containing a single component type, the system signature and survival signature have the simple relation

$$q_l = \Phi_{m-l} - \Phi_{m-l-1} \tag{6}$$

Several theoretical applications of the survival signature to problems in the field of reliability engineering have already been published including nonparametric predictive inference for system reliability [7]; Bayesian inference for reliability of systems and networks [8]; modelling uncertain aspects of system dependability [9]; predictive inference for system reliability after common-cause component failures [10]; imprecise system reliability and component importance [11]; Bayesian nonparametric system reliability using sets of priors [12] and comparing systems with heterogeneous components [13].

Despite the advances is the theory and development of numerous application for signatures, practical applications have until now been limited to the analysis of relatively small problems. The main reason for this is that the computation of signatures using existing methods is difficult unless the number of components is small or the system reliability structure function is quite trivial [2,6]. The aim of this paper is to present a new and computationally efficient algorithm based on binary decision diagrams for computing exact system and survival signatures and report its computational efficiency for a number of example systems, including large and complex systems that have been derived from practice and were published as benchmarks in the literature. The remainder of this paper is organised as follows: Section 2 describes the existing methods that are available for the computation of system and survival signatures. Section 3 introduces the new algorithm. Section 4 presents some results on the efficiency of the new algorithm in computing system and survival signatures for a set of example systems, including some large and complex benchmark systems from the literature. Section 5 summarises the paper, gives some concluding remarks and also discusses limitations and areas for future work.

2. Existing methods for system and survival signature computation

A small number of methods for computing system signatures have been published in the literature and are based on minimal ordered cut sets, diagonal sections of the reliability structure function and generating functions. Kochar et al. [14] note that the system signature can be defined for $j \in \{1,2,\ldots,m\}$ as

$$q_j = \frac{numberofcomponentorderingsforwhichthejthfailurecausessystemfailure}{m!} \tag{7}$$