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The effectiveness of adding cold standby redundancy to a coherent system at system and component levels

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ABSTRACT

The effect of adding cold standby redundancy to a system at system and component levels provides a useful information in reliability design. For a series (parallel) system adding cold standby redundancy at the component (system) level yields longer system lifetime. In this paper, the effect of adding cold standby redundancy to a general coherent structure at system and component levels is studied. In particular, signature-based expressions for the survival function of the system after standby redundancy at system and component levels are obtained. Thus for a given coherent structure with known signature, the survival functions and mean time to failure of new systems can be easily calculated and comparisons can be done in terms of stochastic ordering, and mean time to failure ordering. As a case study, circular consecutive-k-out-of-n:G system which can be used to analyze activities in a nuclear accelerator is considered.

1. Introduction

Cold-standby sparing is an effective method that has been widely used to enhance the reliability of a system in various applications including telecommunication systems, satellites, and nuclear power plants. The study of systems that contain cold standby redundancy has attracted a great deal of attention in reliability literature. Wang et al. [1] presented an approximation model which is based on the central limit theorem for the reliability analysis of binary cold-standby systems. Ardakan and Hamadani [2] studied reliability-redundancy allocation problem with cold-standby redundancy strategy. Levitin et al. [3] considered the optimal standby component sequencing problem for 1-out-of-N:G heterogeneous cold-standby systems. Levitin et al. [4] studied the optimal choice of productivity of components in 1-out-of-N non-repairable cold standby systems. Recently, Wang et al. [5] performed interval analysis for solving cold-standby system reliability optimization problems by considering parameter uncertainty. Coherent systems equipped with a single standby component have been considered in [6–9].

In this paper, the effectiveness of adding standby redundancy to a coherent system at system and component levels is studied. The main objective of this paper is to determine which type of standby redundancy is superior to the other for a coherent system of n active components and equipped with n independent spares. This problem has been formerly treated in the literature for series and parallel systems [10–12]. It has been shown that the system lifetime is longer when standby redundancy is added at the component level for a series system. For a parallel system, standby redundancy at the system level is more efficient. For a design engineer, it is very useful to know the type of standby redundancy which will enhance the reliability larger. Since real life systems are more complex than series and parallel systems, the problem is worthy of investigation for a general coherent structure. In the present paper, this problem is considered by utilizing the concept of signature which only depends on the structure of the coherent system.

The paper is organized as follows. In Section 2, basic definitions, assumptions, and the notation that will be used throughout the paper are presented. Section 3 contains signature-based expressions for the survival function and mean time to failure value of a coherent system after standby redundancy. In Section 4, mean time to failure values of all coherent systems with three and four components after standby redundancy at system and component levels are computed. Section 5 is devoted to the study of circular consecutive-k-out-of-n:G systems.

2. Definitions, assumptions and notation

Consider an arbitrary binary coherent system with n components. A system is said to be coherent if its structure function $\phi: \{0, 1\}^n \longrightarrow \{0, 1\}$ is nondecreasing in each argument, and each component is relevant to the performance of the system. The system consists of independent binary components. Assume that such a system is equipped with n spares which have the same lifetime distribution with active components after activation. These n spares are assumed to be cold standby, i.e. they do not fail while they are in the standby mode. In other words, the hazard rate of a component in the standby mode is

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zero. The switching mechanism is assumed to be fully reliable. Throughout the paper, the following notation will be used.

T_i lifetime of component i in a coherent system, $i = 1, \ldots, n$,
T_i^* lifetime of standby component i, $i = 1, \ldots, n$,
$\phi(T_1, \ldots, T_n)$ lifetime of coherent system with component lifetimes $T_1, \ldots, T_n$,
T_S system lifetime after standby redundancy at system level,
T_C system lifetime after standby redundancy at component level,
MTTF_S mean time to failure of the system after standby redundancy at system level,
MTTF_C mean time to failure of the system after standby redundancy at component level,
$T_{i:n}$ the ith smallest among $T_1, \ldots, T_n$,
$\mathbf{s} = (s_1, \ldots, s_n)$ the signature of the system with lifetime $\phi(T_1, \ldots, T_n)$,
$F(t)$ the cumulative distribution function of T_is and T_i^* s, $i = 1, \ldots, n$,

Standby redundancy can be applied at system or component levels. The lifetime of the system after standby redundancy at system level can be defined as

$$T_S = \phi(T_1, \ldots, T_n) + \phi(T_1^*, \ldots, T_n^*). \quad (1)$$

Similarly, if the standby redundancy is applied at component level, then system's lifetime becomes

$$T_C = \phi(T_1 + T_1^*, \ldots, T_n + T_n^*). \quad (2)$$

For a design engineer, it is very important to elicit information about ordering relation between (1) and (2). The engineer deserves to have an answer to the following question: Which type of standby redundancy (system or component level) yields a longer lifetime for an n component system that is equipped with n standby components? As shown in [10,11], for a series system $T_C \geq_{st} T_S$ while $T_S \geq_{st} T_C$ for a parallel system, where $\geq_{st}$ represent usual stochastic ordering ($X \geq_{st} Y$ if and only if $P\{X > t\} \geq P\{Y > t\}$ for all $t \geq 0$). The lifetime X will be said to be larger than the lifetime Y in MTTF ordering (denoted by $X \geq_{MTTF} Y$) if $E(X) \geq E(Y)$. Clearly, if $X \geq_{st} Y$ then $E(X) \geq E(Y)$.

Consider the system with lifetime

$$T = \phi(T_1, T_2, T_3) = \min(T_1, \max(T_2, T_3)). \quad (3)$$

Fig. 1a and b respectively depict new systems with lifetimes T_S and T_C.

For this system,

$$\begin{aligned} T_C &= \min(T_1 + T_1^*, \max(T_2 + T_2^*, T_3 + T_3^*)) \\ &= \min(T_1 + T_1^*, \max(T_2, T_3) + \max(T_2^*, T_3^*)) \\ &\geq_{st} \min(T_1, \max(T_2, T_3)) + \min(T_1^*, \max(T_2^*, T_3^*)) \\ &= T_S \end{aligned}$$

which implies that the component level standby redundancy is more effective than the system level standby redundancy. Such a simple comparison may not be possible for an arbitrary coherent system, especially when n is not small. In the next section, we compute and compare $P\{T_S > t\}$ and $P\{T_C > t\}$ for a given coherent structure by utilizing the concept of signature.

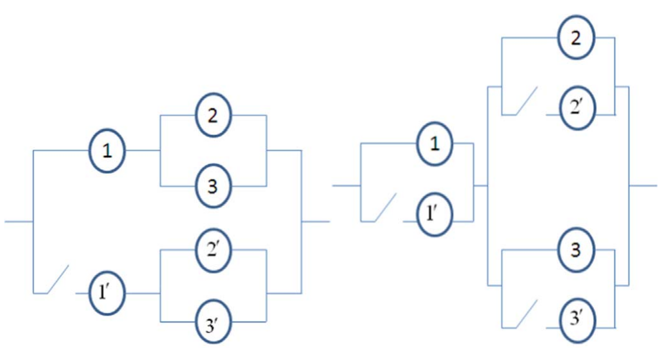

Fig. 1. (a) System level redundancy, (b) Component level redundancy.

3. Signature-based expressions for survival functions

Consider a coherent system with lifetime $T = \phi(T_1, \ldots, T_n)$. If the system consists of independent and identical components having common absolutely continuous lifetime distribution, then its survival function can be represented as

$$P\{T > t\} = \sum_{i=1}^{n} s_i P\{T_{i:n} > t\}, \quad (4)$$

where $s_i = P\{T = T_{i:n}\}$, i=1,...,n. The vector $\mathbf{s} = (s_1, \ldots, s_n)$ has been called the system signature (Samaniego [13]). The signature of a system does not depend on the distribution of $T_1, \ldots, T_n$. The signatures of series and parallel systems of n components are given respectively as $\mathbf{s} = (1, 0, \ldots, 0)$, and $\mathbf{s} = (0, \ldots, 0, 1)$. For the system defined by the lifetime (3), the signature is $\mathbf{s} = (\frac{2}{6}, \frac{4}{6}, 0)$.

The survival function of $T_{i:n}$ can be computed from

$$P\{T_{i:n} > t\} = \sum_{m=n-i+1}^{n} \binom{n}{m} (\bar{F}(t))^m (F(t))^{n-m}, \quad (5)$$

where $F(t) = P\{T_i \leq t\}$ [14].

Eq. (4) is a signature-based expression for the survival function of a coherent system without standby components. Below, signature-based expressions for the survival functions of T_S and T_C are obtained. Throughout the paper it is assumed that $(T_1, \ldots, T_n) \stackrel{d}{=} (T_1^*, \ldots, T_n^*)$ which implies that n spares have the same lifetime distribution with n original components.

Clearly,

$$\begin{aligned} P\{T_S > t\} = &P\{\phi(T_1, \ldots, T_n) > t\} \\ &+ \int_0^t P\{\phi(T_1^*, \ldots, T_n^*) > t - x\} dP\{\phi(T_1, \ldots, T_n) \leq x\}. \end{aligned}$$

Thus, the usage of (4) yields

$$P\{T_S > t\} = \sum_{i=1}^{n} s_i P\{T_{i:n} > t\} + \sum_{i=1}^{n} \sum_{j=1}^{n} s_i s_j \int_0^t P\{T_{i:n}^* > t - x\} f_{j:n}(x) dx, \quad (6)$$

where $P\{T_{i:n} > t\} = P\{T_{i:n}^* > t\}$ is given by (5), and

$$f_{j:n}(x) = \frac{n!}{(j-1)!(n-j)!} (F(x))^{j-1} (1 - F(x))^{n-j} f(x). \quad (7)$$

If the standby redundancy is applied at component level, then the survival function of the system becomes

$$\begin{aligned} P\{T_C > t\} &= P\{\phi(T_1 + T_1^*, \ldots, T_n + T_n^*) > t\} \\ &= \sum_{i=1}^{n} s_i P\{Z_{i:n} > t\}, \end{aligned}$$

where $Z_{1:n} < \ldots < Z_{n:n}$ are ordered lifetimes corresponding to $Z_1 = T_1 + T_1^*, \ldots, Z_n = T_n + T_n^*$. The survival function of Z_i can be written as

$$\bar{G}(t) = \bar{F}(t) + \int_0^t \bar{F}(t - x) dF(x). \quad (8)$$

Thus

$$P\{T_C > t\} = \sum_{i=1}^{n} s_i \sum_{m=n-i+1}^{n} \binom{n}{m} (\bar{G}(t))^m (G(t))^{n-m}, \quad (9)$$

Because $(T_1, \ldots, T_n) \stackrel{d}{=} (T_1^*, \ldots, T_n^*)$, we have $E\phi(T_1, \ldots, T_n) = E\phi(T_1^*, \ldots, T_n^*)$. Thus the MTTF of the system after standby redundancy at system level can be computed from

$$\begin{aligned} MTTF_S &= E(T_S) = E[\phi(T_1, \ldots, T_n) + \phi(T_1^*, \ldots, T_n^*)] \\ &= 2E\phi(T_1, \ldots, T_n) \\ &= 2\sum_{i=1}^{n} s_i E(T_{i:n}), \end{aligned} \quad (10)$$