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On sensitivity analysis of aging multi-state system by using L_Z-transform

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ABSTRACT

The paper considers a sensitivity evaluation for an aging multi-state system (MSS) under minimal repair. Investigation of an impact of changing different failure/repair rates of different elements in MSS is important for practical reliability engineering. In practical reliability engineering a "curse of dimensionality" (the large number of states that should be analyzed for a multi-state system model) is a main obstacle for sensitivity assessment. Straightforward Markov Method applied to solve this problem requires building a model with numerous numbers of states and solving a corresponding system of differential equations. In order to solve this problem, the paper proposes to use a new method based on an L_Z-transform of the discrete-state continuous-time Markov process, and on Ushakov's Universal Generating Operator. New sensitivity measures useful for aging MSS reliability analysis were introduced. It was shown that the proposed method drastically reduces a computational burden. A numerical example is presented in order to illustrate the approach.

1. Introduction

Sensitivity analysis is used to ascertain how a given model output depends upon the input parameters. This is an important method for checking the quality of a given model, as well as a powerful tool for checking the robustness of its analysis. The paper [16] investigates reliability of a repairable system with imperfect coverage under service pressure condition. The sensitivity analysis and relative sensitivity analysis of reliability characteristics are used to study the effects of different parameters on the system reliability and mean time to failure. In the research [8], two factorial design experiments based on the cost associated with maintenance and replacement activities and reliability characteristic parameters are constructed and analyzed. Experimental results of a sensitivity analysis on the optimization models are presented and evaluated. These experiments investigate the effect of the parameters on the structure of optimal preventive maintenance and replacement schedules in multi-component systems. The paper [2] deals with modeling the movement and consequence of radioactive pollutants that are critical for environmental protection and control of nuclear facilities. Sensitivities of 21 input parameters have been analyzed for a specific-activity tritium dose model using fourteen methods of parameter sensitivity analysis. In [11] are carefully described principles of sensitivity analysis and given suitable methods for approaching many types of problems. A most common approach for sensitivity analyses is changing one factor at a time (OAT) [11], to see what effect this has on the output. OAT involves the following steps: (a) moving one input variable, keeping others at their baseline (nominal) values; (b) returning the variable to its nominal value, and repeating for each of the other inputs the same procedure. Sensitivity may then be measured by monitoring changes in the output by partial derivatives.

Investigation of an impact of changing different failure rates in different elements in multi-state system (MSS) is often important for practical reliability engineering. Based on this reliability a researcher or an engineer can make appropriate decisions for MSS reliability improvement.

When we are dealing with MSS, the most important reliability measures are a system availability, a mean MSS output performance, a mean output performance deficiency, and a mean accumulated performance deficiency [7].

For aging MSS such parameters as availability and mean output performance are usually decreasing over time t, and accumulated performance deficiency is increasing.

In practice, reliability engineer is often interested in estimating the impact of changing of different system parameters such as failure and repair rates of different MSS's components to the entire system

Abbreviation: MSS, multi-state system; OAT, one factor at time; UGF, universal generating function; DSCT, discrete- state continuous-time; PMS, phased-mission system; UGO, universal generating operator

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Nomenclature

n number of components in MSS;
k_j number of possible states for component j.
g_{ji} performance of component j in the state i, $i \in \{1, 2, ..., k_j\}$.
$\mathbf{g}_j = \left\{g_{j1}, ..., g_{jk_j}\right\}$ set of possible states of component j;
$\mathbf{A}_j = (a_{lm}^{(j)}(t)),\ l,\ m = 1, ..., k_j;\ j = 1, ...n$ transition intensities matrix of component j;
$p_{ji}(t)$ probability that at time instant t component j is in state j;
$G_j(t) \in \mathbf{g}_j,\ j = 1, \ldots, n$ stochastic performance process of component j;
f system structure function; operator Ω_f
Ω_f Ushakov's universal generating operator for MSS with structure function f;
$G(t) = f(G_1(t), ..., G_n(t))$ output performance stochastic process of entire system;
$L_Z\{G(t)\}$ L_Z-transform of stochastic process $G(t)$;
$A_w(t)$ MSS instantaneous availability at time instant t for required demand level w;
A_O specified (required) level of availability;
$T_{A_w \ge A_0}$ time up to the instant, when MSS availability will down cross lower level A_O;
$T_{\overline{G} \ge G_0}$ time up to the instant, when mean value of stochastic process $G(t)$ will down cross some preliminary specified level G_0;
$T_{D_w \le D_0}$ time up to the instant, when accumulated performance deficiency $D_w(t)$ will be greater than previously specified level D_0;
$S_{a_{lm}^{(j)}}^{A_w} = \frac{\partial T_{A_w \ge A_0}}{\partial a_{lm}^{(j)}}$ sensitivity measure for aging MSS lifetime with respect to availability restriction;
$S_{a_{lm}^{(j)}}^{\overline{G}} = \frac{\partial T_{\overline{G} \ge G_0}}{\partial a_{lm}^{(j)}}$ sensitivity measure for aging MSS lifetime with respect to mean output performance restriction;
$S_{a_{lm}^{(j)}}^{D_w} = \frac{\partial T_{D_w \le D_0}}{\partial a_{lm}^{(j)}}$ sensitivity measure for aging MSS lifetime with respect to accumulated performance deficiency restriction;

parameters.

A main obstacle in multi-state system availability assessment is what is often referred to as a "curse of dimensionality". For an aging MSS with number of different components this obstacle is even critical and sometimes avoids application of traditional Markov methods. In order to overcome this problem a specific approach called the Universal Generating Function (UGF) technique has been introduced and then successfully applied to MSS reliability analysis. The UGF technique allows one to find algebraically an entire MSS performance distribution through the performance distributions of its elements. The basic ideas of the method were primarily introduced by Ushakov [14] in the mid-1980s. Since then, the method has been considerably expanded in numerous research works. In the books [4] and [7] one can find a historical overview, detailed UGF-method description and the method applications to various important practical cases. In [5] UGF method was applied to importance and sensitivity analysis of MSS. The main limiting factor of the UGF technique's application to real-world MSS reliability analysis is the fact that UGF is based on a moment generating function that is mathematically defined only for **random variables**. This fact is the reason to consider performance of each MSS's element as a random variable, in spite of the fact that in reality it is a discrete-state continuous-time stochastic (DSCT) process [7,9]. In practice, it means that only steady state behavior of MSS can be analyzed and, so, aging MSS is out of the scope. In fact, some modification of UGF method was suggested in papers [10] and [12] in order to overcome this obstacle. It was proven that UGF is not only useful for steady state systems, but also can be applied for phased-mission systems (PMS), which can also model aging MSS. However, PMS does not use continuous time model, but divides the time into several phases. In general case, when there are many aging components in the system with different increasing failure rates, it may be difficult.

In order to remove this restriction a special transform that was called L_Z-transform was introduced in [6] for discrete-state continuous-time Markov process. This transform allows the Ushakov's Universal Generating Operator Ω_f to be applied. Such important properties as existence and uniqueness were proven for L_Z-transform. It was shown that many important MSS's reliability measures such as instantaneous availability $A(t)$, mean instantaneous performance, mean instantaneous performance deficiency, etc., can be found by using L_Z-transform.

In this paper we suggest the method for aging MSS sensitivity analysis, which is based on L_Z-transform application. It will be shown that by using the method one can avoid building a Markov model for entire MSS and solution of corresponding system with great number of differential equations. Instead, one can solve only simple models for the system's elements and then use a simple algebra.

2. Sensitivity analysis for aging MSS by using L_Z-transform method

2.1. MSS model for the method application

We consider a multi-state system consisting of n multi-state components. Any component j in MSS can have k_j different states corresponding to different performances, represented by the set $\mathbf{g}_j = \left\{g_{j1}, ..., g_{jk_j}\right\}$, where g_{ji} is the performance of component j in the state i, $i \in \{1, 2, ..., k_j\}$.

Usually the states are ordered in according to their performances. The generic MSS model consists of the performance stochastic processes $G_j(t) \in \mathbf{g}_j,\ j = 1, \ldots, n$ for each system component j, and the system structure function f that produces the stochastic process corresponding to the output performance of the entire MSS: $G(t) = f(G_1(t), ..., G_n(t))$.

At first, a model of stochastic process should be built for every multi-state component. Markov performance stochastic process for each component j can be represented by the triplet $G_j(t) = \left\{\mathbf{g}_j, \mathbf{A}_j, \mathbf{p}_{j0}\right\}$, where $\mathbf{g}_j, \mathbf{A}_j, \mathbf{p}_{j0}$ are defined by the following:

$\mathbf{g}_j = \left\{g_{j1}, ..., g_{jk_j}\right\}$ - set of possible component's states;

$\mathbf{A}_j = (a_{lm}^{(j)}(t)),\ l,\ m = 1, ..., k_j;\ j = 1, ...n$ - transition intensities matrix. For components with aging properties some matrix's elements $a_{lm}^{(j)}(t)$ are increasing functions of time, for non-aging components all matrix's elements are constant. For further application of L_Z-transform it is important that all elements of the matrix are continuous function of time.

$\mathbf{p}_{j0} = \left[p_{10}^{(j)} = \Pr\left\{G_j(0) = g_{j1}\right\}, ..., p_{k_j0}^{(j)} = \Pr\left\{G_j(0) = g_{jk_j}\right\}\right]$ - initial states probability distribution.

For each component j the following system of differential equations can be written for the state probabilities [13]: