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New interval availability indexes for Markov repairable systems

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ABSTRACT

In maintenance field, there are many indexes such as instantaneous availability, steady state availability, interval availability and others, which have been widely used to describe the various properties and performance of repairable systems on maintenance. However, all the present availability indexes still cannot cover some situations which people are interested in. In the paper, two new interval availability indexes are introduced for Markov repairable systems, which is named as an availability with a given window length and containing a specified point or interval. The new interval availability is the probability that the system is working during a given window length and this window must contains a specified point or interval. Their calculation formulas are presented in matrix forms by using the Laplace transform. Some properties on the two indexes are discussed briefly, and numerical examples are shown to illustrate the features of the two new indexes, finally conclusions are given. These two new indices can become the conventional interval availability when the given window length is zero or equal to the specified interval length. The results in the paper may be used to measure the performance of repairable system more deeply and detail.

1. Introduction

In reliability field, the related measure indexes such as reliability, availability and safety etc. play an important role to describe system performances. Any theoretical research and engineering work in reliability must be related to the reliability measure indexes because these measure indexes can tell people if they have reached the pre-specified target or if the state of system operating can meet the mission requirements. On the other hand, each system including that in reliability field has many performances, how to measure these performances of the system must rely on some indexes. As the technology developments, people want to know more deep and detail information on systems, new indexes are needed naturally. The similar situation in maintenance is faced at present, some new measure indexes are needed to describe the situation or information people are interested in. In fact, many indexes have been used in describing the various properties of repairable systems. For example, several reliability indexes for repairable systems were proposed in [1–3]. The reliability and availability indexes are an important topic, much literature can be found in this direction, for example, references [4,5]. Specially, on aggregated repairable systems, many availability indexes including the conventional pointwise and interval availabilities have been studied, for example, references [6–10]. The availability indexes are closely related to maintenance models because the maintenance models can describe the evolution processes of the repairable systems, while the availability indexes can describe the performance levels of the repairable systems under given models. For example, Cui et al. [11] introduced a new changeable state repairable system, and then the conventional availability indexes were given in their paper. Recently, Liu et al. [12] and Cui et al. [13] studied some availability indexes under aggregated stochastic models. In fact, it is not an easy task to give the formulas for these indexes, and the related work is mainly for single-unit systems, Markov repairable systems and semi-Markov repairable systems, for example, Refs. [14–16].

Availability is an important measure of performance for repairable systems, which has been developed into more detail indexes such as pointwise availability, interval availability and joint availability and so forth. In general, the availability is a probability that the repairable system will be able to operate within the tolerances at a given instant time or interval, which are for point-wise and interval availabilities. The definition of interval availability has some different ways. The interval availability commonly refers to the expected proportion of time for which the repairable system is available over some interval of time. Sericola [17] defined it as the fraction of operation time over a finite observation period, which is a random variable. Finkelstein [18] defined a multiple availability as the probability of a system being in an operating state at each moment of demand. Csenki [19] also studied multiple availability, but in his work multiple availability was called joint availability. Csenki [20,21] also studied the related interval availability. Cui et al. [22] proposed the multi-point, multi-interval,

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and mixed multi-point-interval availabilities, which are extensions of the single point and interval availabilities. A similar concept is an interval reliability, Barlow and Proschan [23] defined it as the probability that at a specified time the system is operating and will continue to operate for an interval of duration. (The authors would like to call this interval reliability as interval availability not interval reliability especially for repairable systems, although the classical book and mathematical reliability pioneers named it. On the other hand, the name change for interval reliability can meet the feature of the pointwise availability, multiple availability and repairable systems, because when the interval length becomes into zero, multiple into single, the interval reliability becomes into the pointwise availability, but the multi-interval availability become into interval reliability.). Wu and Hillston [24] studied two kinds reliability measures under a semi-Markov process situation, one is the system must remain operational continuously for a minimum time within the given mission time interval, while the other required the total operational time of the system within the mission time window must be greater than a given value. Both reliability measures are defined by suing the fraction of working time to window length, and the first required the continuous working time, but the second is not required. As mentioned previously, there are many indexes for repairable systems, but they still cannot cover all practical situations, For example, people are interested in the probability that the system, which may be a computer system or an air condition system or service system etc., is available (working) at 9:00 o'clock or so within 15 min, or the probability that the system is available (working) during a window interval whose interval length is a half hour but it must cover the interval from 9:00 o'clock to 9:15 o'clock. This kind of problem is what our present paper will study, we name the first probability as an availability with window τ containing point x, which is the probability that a repairable system works throughout an interval window which has at least a length τ and contains a given point x. The second probability is called an availability with window τ containing interval $[a, b]$, which is the probability that a repairable system works throughout an interval window which has at least a length τ and contains a given interval $[a, b]$. Both concepts are extensions of pointwise availability and interval availability, which definitely enrich the indexes measures in maintenance. For application examples of two indices introduced above, we can look at a water supply system. The requirement of customers on this system is: it must be working from 9:00 o'clock to 9:15 o'clock, but the manager wants to guarantee this customer requirement to be satisfied more likely, he/she requires the water supply system must be working at least a half hour which must also cover the time interval [9:00, 9:15]. This management strategy can provide more tolerance for the customer requirement. The author believe that the two new availability indices can be used in more real situations, particularly in increasing mission success probability of systems.

In the paper, after doing the introduction of two new availability indexes, their calculation formulas are given by using the Laplace transform and matrix techniques, meanwhile, some basic results referenced in Colquhoun and Hawkes [25] are used in the process of derivations. Furthermore, some properties of the two new availability indexes are discussed briefly, which may provide more information and understanding on usage of the two new availability indexes.

The rest of the paper is organized as follows. Section 2 provides the assumptions on Markov repairable model and definitions of two new availability indexes, including their mathematical expressions, and some basic results referenced in Colquhoun and Hawkes [25] are presented. In Section 3, the main results of the paper, the calculation formulas of the two new availability indexes, are given. Some properties and comparisons of the two new availability indexes are considered in Section 4, which includes the relationship between the two new availability indexes and the conventional pointwise and interval availabilities. Section 5 presents some numerical examples to illustrate the properties of the two new availability indexes, which may provide some intuitive understanding on usage of the two new availability indexes. Finally, the conclusions are summarized in Section 6.

2. Assumptions, definitions and preliminaries

Suppose there is a repairable system following a homogeneous continuous time Markov process $\{X(t), t \geq 0\}$ with finite state space S which contains a working subset and a failure subset, i.e., $S = W \cup F$, where $S = \{1, 2, \ldots, n\}$ and $F = \{n + 1, n + 2, \ldots, n + m\}$. The infinitesimal generator of the process $\{X(t), t \geq 0\}$ is $\mathbf{Q}$, in terms of working and failure states, which can be divided into four blocks, i.e.

$$\mathbf{Q} = \begin{pmatrix} \mathbf{Q}_{WW} & \mathbf{Q}_{WF} \\ \mathbf{Q}_{FW} & \mathbf{Q}_{FF} \end{pmatrix}.$$

The definitions of two new availability indexes to be discussed throughout the paper are given as follows.

Definition 1. The probability that a repairable system works throughout an interval window, which has at least a length τ and contains a given point x, is called as an availability with window τ containing point x.

The availability with window τ containing point x can be expressed in a formula way as

$$A(\tau, x) := P\{\exists c \geq 0, \text{ system works in interval } [c, c + \tau] \text{ and } c \leq x \leq c + \tau\},$$

where τ is a required interval length, x is a given instant.

The availability with window τ containing point x can be used to describe an interval availability in which the interval must have at least a length τ and contain a given time instant x, it is an extension of point availability. When the interval length $\tau = 0$, the availability with window τ containing point x becomes into a conventional point availability at time instant x. To understand the concept of availability with window τ containing point x, we can image that there is a window with length τ to move towards right or left containing point x and at least at one moment the repairable system works throughout the window, the probability for this situation is the availability with window τ containing point x.

Definition 2. The probability that a repairable system works throughout an interval window which has at least a length τ and contains a given interval $[a, b]$, is called as an availability with window τ containing interval $[a, b]$.

The availability with window τ containing interval $[a, b]$ can be expressed in a formula way as

$$A(\tau, [a, b]) := P\{\exists c \geq 0, \text{ system works in interval } [c, c + \tau] \text{ and } c \leq a \leq b \leq c + \tau\},$$

where τ is a required interval length, $[a, b]$ is a given interval.

Similarly, the availability with window τ containing interval $[a, b]$ can be used to describe an interval availability in which the interval must have at least a length τ and contain a given interval $[a, b]$, this new availability concept is an extension of interval availability. When the interval length $\tau = b - a$, the availability with window τ containing interval x becomes into a conventional interval availability at interval $[a, b]$. To understand the concept of availability with window τ containing point $[a, b]$, we can image that there is a window with length τ to move towards right or left covering interval $[a, b]$, and at least at one moment the repairable system works throughout the window, the probability for this situation is the availability with window τ containing interval $[a, b]$.

Because of using Laplace transform throughout the paper, here we give its definition below and specify the Laplace transform on matrix for elementwise transforms.

The Laplace transform for function $f(t)$ is defined as follows,