Reliability Engineering and System Safety 168 (2017) 40–52

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

On geometric reduction of age or intensity models for imperfect maintenance

Laurent Doyen[a], Olivier Gaudoin[a,*], Annamraju Syamsundar[b]

[a] Univ. Grenoble Alpes, LJK, CNRS, F-38000 Grenoble, France
[b] Research and Development Centre, Visakhapatnam Steel Plant, Rashtriya Ispat Nigam Limited, Visakhapatnam, India

ARTICLE INFO

Keywords:
Repairable systems reliability
Imperfect maintenance
Ageing
Virtual age

ABSTRACT

There is an extensive literature on imperfect maintenance models. Among them, we have proposed in Doyen and Gaudoin (2004) [13] the classes of ARA and ARI models. These models express the effect of maintenance by an Arithmetic Reduction of Age (for ARA models) and an Arithmetic Reduction of Intensity (for ARI models). The aim of this paper is to study the possibility of building equivalent models with a Geometric Reduction of Age or Intensity: the GRA and GRI models. Several ideas leading to different possible models are presented. A list of potential GRI-GRA models is established. A brief illustration of their behavior is given based on simulated data and an application to real maintenance data is presented.

1. Introduction

The reliability of a repairable system depends on both the ageing process and the effectiveness of maintenance actions. In this paper, we consider only corrective maintenance or repair, but the same kind of problems can be studied when both corrective and preventive maintenances are considered. The basic assumptions on maintenance effectiveness are known as minimal maintenance or As Bad As Old (ABAO) effect and perfect maintenance or As Good As New (AGAN) effect. The corresponding stochastic models for the failure process are respectively the Non Homogeneous Poisson Processes (NHPP) and the Renewal Processes (RP). Reality is generally between these two extreme cases: a standard maintenance is better than minimal but not necessarily perfect. This is known as imperfect maintenance.

Many imperfect maintenance models have been proposed. All these models are defined by their assumptions on intrinsic ageing and maintenance effect. Among them, some of the most usual models are the Brown-Proschan [3], Virtual Age [15] and Geometric Process [16] models.

In [12], we have proposed two classes of models, the ARA and ARI models, for which the effect of maintenance is expressed respectively by an Arithmetic Reduction of virtual Age and an Arithmetic Reduction of failure Intensity. The ARA models, and to a lesser extent the ARI models, have been widely used [18,8,20,23,9,10,21] and have been proved to provide a good fit for many real maintenance data sets. In [12], we said that *other types of reduction such as geometric reduction can be considered*, but this idea was not developed at that time. At first look, using a geometric reduction of age or intensity is an attractive prospect because it opens the possibility of considering behaviors in the failure process which cannot be tackled by ARA and ARI models, such as a stronger slowdown of the wear. Moreover, this idea could provide a trade-off between ARA-ARI models and the Geometric Process.

So the aim of this paper is to study the possibility of building imperfect maintenance models with a Geometric Reduction of Age or Geometric Reduction of Intensity: the GRA and GRI models. For the sake of simplicity, we will consider only the equivalent of ARA_1 and ARI_1 models. An unexpected result of this study is that it revealed a problem that may happen in some cases with the usual definition of the ARI_1 model. So an alternate definition of the ARI_1 model is proposed.

Section 2 recalls the general framework on imperfect maintenance modelling and briefly describes the most usual models. Section 3 presents in more details the ARA and ARI models. In Sections 4 and 5, several ideas leading to different possible GRI-GRA models are presented. A list of potential models is established, in which the models are defined by their failure intensity. The models derived from a direct adaptation of the ARI-ARA assumptions are presented in Section 4. Since this approach appears to be not satisfactory, other propositions are given in

Abbreviation: ABAO, As Bad As Old; AGAN, As Good As New; AIC, Akaike Information Criterion; ARA, Arithmetic Reduction of Age; ARI, Arithmetic Reduction of Intensity; BP, Brown-Proschan; CP, Calabria-Pulcini; GFRR, Geometric Failure Rate Reduction; GP, Geometric Process; GRA, Geometric Reduction of Age; GRI, Geometric Reduction of Intensity; LLP, Log Linear Process; NHPP, Non Homogeneous Poisson Process; PLP, Power Law Process; QR, Quasi-Renewal; RP, Renewal Process.

* Corresponding author.
E-mail addresses: laurent.doyen@imag.fr (L. Doyen), olivier.gaudoin@imag.fr (O. Gaudoin), syamsundar.annamraju@gmail.com (A. Syamsundar).

http://dx.doi.org/10.1016/j.ress.2017.03.015
Available online 12 April 2017
0951-8320/

Section 5. Some experimental results are presented in Section 6, based on simulated and real maintenance data. Finally, Section 7 draws some conclusions.

2. Imperfect maintenance models

2.1. General framework

Let $\{T_i\}_{i\geq 1}$ be the successive failure times of a repairable system, starting from $T_0 = 0$. We assume that a corrective maintenance, or repair task, is performed after each failure. Repair durations are assumed to be negligible, so the failure and repair times can be confounded. Let $\{X_i\}_{i\geq 1}$ be the times between failures ($X_i = T_i - T_{i-1}$) and N_t be the number of failures observed up to time t. Failure times are recurrent events and, as usual in this field [7], it is assumed that two failures cannot occur simultaneously.

Then, the failure process is a random point process. Its distribution can be characterized by the failure intensity, defined as [1]:

$$\forall t \geq 0, \quad \lambda_t = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P(N_{(t+\Delta t)-} - N_{t-} = 1 | \mathcal{H}_{t-}) \tag{1}$$

where $\mathcal{H}_{t-}$ is the history of the failure process just before time t, i.e. the set of all events occurred just before t. N_{t-} denotes the left-hand limit of N_t.

In most cases, the failure process is a self-excited process, i.e. λ_t is a function of the number of failures and the failure times before t: $\lambda_t = \lambda_t(N_{t-}; T_1, \ldots, T_{N_{t-}})$. In this case, the distribution of the failure process is completely given by the intensity. Self-excited point processes is a wide class of point processes including Poisson processes, renewal processes, Hawkes processes and most imperfect maintenance models.

We assume that before the first failure, the intensity is a deterministic continuous function of time, denoted $\lambda(t)$, and called the initial intensity. The initial intensity characterizes the intrinsic behavior of a new unmaintained system. $\lambda(t)$ is the hazard rate of the first failure time $T_1 = X_1$. Note that the failure intensity λ_t is a stochastic process while the initial intensity $\lambda(t)$ is a deterministic function of time. The failure intensity and the initial intensity are equal before the first failure, but after that they are different, except in the NHPP case (see below).

The usual assumptions on the initial intensity are:

- Power Law Process (PLP): $\lambda(t) = \alpha\beta t^{\beta-1}, \alpha > 0, \beta > 0$.
- Log-Linear Process (LLP): $\lambda(t) = \exp(\alpha + \beta t), \alpha \in \mathbb{R}, \beta \in \mathbb{R}$.

In this paper, we will consider systems which are ageing intrinsically, so $\lambda(t)$ is increasing. Then $\beta > 1$ for the PLP and $\beta > 0$ for the LLP.

A convenient way to define an imperfect maintenance model is to consider that it will be characterized by two features, which will be the model hypotheses:

- H_1: Characterizes the effect of a maintenance just after a failure, expressed by the relationship between the failure intensity after and before T_i, $\lambda_{T_i^+}$ and $\lambda_{T_i^-}$, for each $i \geq 1$.
- H_2: Characterizes the behavior of the system between two successive failures, expressed by the expression of the intensity λ_t in $]T_{N_t}, T_{N_t+1}]$.

In the NHPP case (ABAO), maintenance is minimal so we have:

- H_1: $\forall i$, $\lambda_{T_i^+} = \lambda_{T_i^-}$.
- H_2: The shape of the intensity has not been changed by the maintenance action.

Then, the intensity is simply a function of time:

$$\textbf{NHPP(ABAO)}: \lambda_t = \lambda(t) \tag{2}$$

In the RP case (AGAN), maintenance is perfect and renews the system, so we have:

- H_1: $\forall i$, $\lambda_{T_i^+} = 0$.
- H_2: The intensity after a maintenance is the intensity of a new system.

Then, the intensity is simply a function of the time elapsed since last repair:

$$\textbf{Renewal Process(AGAN)}: \lambda_t = \lambda(t - T_{N_{t-}}) \tag{3}$$

In the following, we briefly describe the most usual imperfect maintenance models. For a more exhaustive review on these models, the reader is referred to [22,23] and the references therein.

2.2. Usual imperfect maintenance models

The Virtual Age models, sometimes called G-Renewal processes, have been proposed by Kijima [15]. They assume that the effect of maintenance is to rejuvenate the system in such a way that the intensity at time t is equal to the initial intensity at time V_t, where the virtual age V_t is less than t. Moreover, they assume that there exists a sequence of random variables $\{A_i\}_{i\geq 1}$, with $A_0 = 0$, such that after the ith repair, the system behaves like a new one having survived without failure until A_i. This leads to the fact that the virtual age of the system at time t is $V_t = A_{N_{t-}} + t - T_{N_{t-}}$. $A_i = V_{T_i}^+$ is the virtual age just after the i^{th} repair and is called the i^{th} effective age. The hypotheses defining a Virtual Age model are then:

- H_1: The effect of the i^{th} repair is defined by the effective age A_i, or equivalently by the relationship between $V_{T_i}^+ = A_i$ and $V_{T_i}^- = A_{i-1} + X_i$.
- H_2: The intensity at time t is the initial intensity at time $V_t = A_{N_{t-}} + t - T_{N_{t-}}$.

So the failure intensity is:

$$\textbf{Virtual Age model}: \lambda_t = \lambda(V_t) = \lambda(A_{N_{t-}} + t - T_{N_{t-}}) \tag{4}$$

The virtual age can also be written:

$$V_t = t - \sum_{i=1}^{N_{t-}} \left[V_{T_i^-} - V_{T_i^+} \right] \tag{5}$$

Between two consecutive failures, the virtual age is parallel to the real age. So the failure intensity of a virtual age model is horizontally parallel to the initial intensity.

The NHPP is a Virtual Age model with $\forall i, A_i = T_i$ and the RP is a Virtual Age model with $\forall i, A_i = 0$.

The Brown-Proschan model [3] assumes that each maintenance is perfect with probability p and minimal with probability $1 - p$. This model is a Virtual Age model (same H_2 as before) and the effective age is:

- H_1: A_i is the time elapsed since the last perfect repair.

Then, the intensity is:

$$\textbf{Brown – Proschan model (BP)}: \lambda_t = \lambda\left(t - T_{N_{t-}} + \sum_{j=1}^{N_{t-}} \left[\prod_{k=1}^{N_{t-}} (1 - B_k) \right] X_j \right) \tag{6}$$

where the B_k's are the successive repair effects: B_k=1 if the kth maintenance is AGAN, and B_k=0 if it is ABAO.

Lam [16] and Wang and Pham [24] proposed independently two models which are equivalent, the Geometric Process (GP) and the Quasi-Renewal Process (QR). These models are based on a proportionality relation between successive times between failures. More precisely, they assume that the times between failures $\{X_i\}_{i\geq 1}$ are independent and that,