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Comparisons of replacement policies with periodic times and repair numbers

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ABSTRACT

Periodic replacement policies modeled with the history of minimal repairs have been studied extensively. However, in the viewpoint of cost rate, there is no literature to compare replacement polices which are carried out at some periodic times and at a predetermined number of repairs. In this paper, we compare these two types of replacement policies analytically from the optimizations of the integrated models. It will be shown that there always exists a degradation model when any bivariate replacement policy is optimized and this is just the best choice of the comparisons. Not only that, the approaches of whichever occurs first and last are applied to model the above two types of policies, which are named as replacement first and replacement last, respectively, and their comparisons are also made. In addition, we delay the policy at repair to periodic time for easier replacement, and the modified replacement model, which is named as replacement overtime, is compared with the original ones. Numerical examples are also given and agree with all analytical discussions.

1. Introduction

In order to be strong enough in production capacity, the total productive maintenance (TPM) has been indispensable in Japanese industry since 1970s, which involves an innovative concept of maintaining equipments by everyone in the organization [1,2]. TPM generally uses periodic and predictive maintenances to aim at maximizations of equipment utilization and production stability. Periodic maintenance [3] consists of periodically inspecting, cleaning and servicing equipment and replacing deteriorated parts to prevent serious breakdown and process problem, while predictive maintenance [4,5] is a method to operate the equipment to the limit of its service life, by measuring and analyzing data about deterioration at routine diagnosis and minor repairs.

Normally, maintenances are more easily to be performed at periodic times in applications, e.g., a complete maintenance in TPM should be carried out on monthly holidays if the equipment must be stopped and a long time is required for maintenance [2]. Theoretical research works on periodic patterns in maintenance plans were studied extensively. Periodical inspection intervals were optimized to detect soft failures of a complex repairable unit, while hard failures create opportunities for additional inspections of all soft-type components [6,7]. Maintenance policies with periodic inspections were observed for the systems with several failures [8] and failure interactions [9]. Inspections are carried out periodically on the unit using the delay time concept during two failure state processes [10]. A k-out-of-n load-sharing unit that is periodically inspected to detect failed components was studied [11]. An optimal number of periodic inspections and its maintenance level to minimize the expected total warranty cost for the second-hand product during the warranty period were derived [12].

In order to achieve just-in-time (JIT) principle in TPM [1,2], repairs are included in maintenance schedules in response to all non breakdown deteriorations and resume quickly the operation of equipment after repairable failure. Not only that, the service life of important part can be predicted based on diagnosis at repairs for predictive maintenance decisions in TPM [3]. Repair models have been studied especially for large and complex systems, which consist of many kinds of units [13]. In recent works, models for repairable unit subjected to minimal repair [14], imperfect repair considering time-dependent repair effectiveness [15], age-based replacement with repair for shocks and degradation [16,17], inspection modeling for repairs [18], post-warranty maintenance with repair time threshold [19], random working models with replacement and minimal repair [20,21], etc., have been studied extensively. More recently, preventive maintenance should be planned jointly with the right type of repairs to achieve the best performance for a multi-state

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unit has been studied [22]. A nonparametric estimation method for periodic replacement problem with minimal repairs has been proposed [23]. A case study of periodic maintenance policy under imperfect repair for off-road engines has been surveyed [24].

It has been well-known in reliability theory that the cost rate model for periodic replacement is formulated by using the cumulative hazard function $H(t)$, where $H(t)$ presents the expected number of failures during [0, t] when the failure rate of a repairable unit remains undisturbed by minimal repairs [25,26]. Recent periodic maintenance models based on the history of failures/repairs can be found extensively in above literatures. However, in the viewpoint of cost rate, there is no study until now to compare replacement polices that are carried out at some periodic times and at a predetermined number of repairs, which becomes the main problem to be discussed in this paper. Furthermore, in order to achieve the target of maximizations of equipment utilization and production stability in TPM [3], the approaches of modelings for replacement first [26], replacement last [27], and replacement overtime [28,29] are used to formulate our replacement models, that is, three approaches of *whichever triggering event occurs first, whichever triggering event occurs last*, and *replacing over a planned measure* will be taken into considerations in replacement policies, respectively. It has been shown in [21,27–29] that replacement last and replacement overtime could let the equipment operate as long as possible, which will also be discussed and compared for the models in this paper.

Our comparisons start from formulating the integrated models with two types of maintenances that are planned at periodic times and at repair numbers. Obviously, when the optimal maintenance policies are obtained in separative models, it will be very easy to compare them with numerical examples in viewpoint of cost rates. However, we compare these two types of maintenances analytically from the optimizations of the integrated models. It will be shown that there always exists a degradation model when any bivariate replacement policy is optimized and it is just the best choice of the comparison, which agrees with the comparisons obtained in [21]. In addition, other comparative results among replacement policies are obtained analytically and numerically.

2. List of assumptions

In this section, the following assumptions for failure, minimal repair and replacement are given:

i. Failures of an operating unit occur at a nonhomogeneous Poisson process with mean value function $H(t) \equiv \int_0^t h(u)\mathrm{d}u$. Let $p_j(t)$ and $P_j(t)$ be the respective probabilities of exact number j of failures and at least number j of failures occur in [0, t], then
$$p_j(t) = \frac{[H(t)]^j}{j!}\mathrm{e}^{-H(t)} \quad \text{and} \quad P_j(t) \equiv \sum_{i=j}^{\infty} p_i(t) \quad (j = 0, 1, 2, \cdots),$$
where $P_0(T) = 1$ and $P_j(\infty) = 1$.

ii. The unit undergoes minimal repairs at failure events, and begins to run again after repairs. It is also assumed that the failure rate $h(t)$ remains undisturbed by minimal repairs, i.e., the unit after each minimal repair has the same failure rate as before failure.

iii. A new unit is replaced at time NT $(N = 1, 2, \cdots; 0 < T < \infty)$, at repair number K $(K = 1, 2, \cdots)$, and at the next periodic time $(n+1)T$ when a number K $(K = 1, 2, \cdots)$ of repairs have been done during $[nT, (n+1)T]$.

iv. The times for minimal repair and replacement are negligible, and let c_R be the cost of replacement and c_m be the cost of minimal repair at each failure, where $c_R \geq c_m$.

3. Replacement first

3.1. Expected cost rate

Suppose that the unit is replaced at a periodic time NT $(N = 1, 2, \cdots; 0 < T < \infty)$ or at a repair number K $(K = 1, 2, \cdots)$, *whichever occurs first*. Then, the probability that the unit is replaced at time NT is $\overline{P}_K(NT)$, and the probability that it is replaced at failure K is $P_K(NT)$. Denoting $\overline{P}_j(t) \equiv 1 - P_j(t)$, and noting that

$$P_j(T) = \int_0^T p_{j-1}(t)h(t)\mathrm{d}t, \quad \overline{P}_j(T) = \int_T^\infty p_{j-1}(t)h(t)\mathrm{d}t,$$
$$\sum_{j=1}^K P_j(T) = \int_0^T \overline{P}_K(t)h(t)\mathrm{d}t, \quad \sum_{j=1}^K \overline{P}_j(T) = \int_T^\infty \overline{P}_K(t)h(t)\mathrm{d}t,$$
$$\sum_{j=K}^\infty P_j(T) = \int_0^T P_{K-1}(t)h(t)\mathrm{d}t, \quad \sum_{j=K}^\infty \overline{P}_j(T) = \int_T^\infty P_{K-1}(t)h(t)\mathrm{d}t,$$

the mean time to replacement is

$$(NT)\overline{P}_K(NT) + \int_0^{NT} t\mathrm{d}P_K(t) = \int_0^{NT} \overline{P}_K(t)\mathrm{d}t, \tag{1}$$

and the expected number of minimal repairs until replacement is

$$\sum_{j=0}^{K-1} jp_j(NT) + KP_K(NT) = K - \sum_{j=0}^{K}(K-j)p_j(NT)$$
$$= \sum_{j=1}^{K} P_j(NT) = \int_0^{NT} \overline{P}_K(t)h(t)\mathrm{d}t, \tag{2}$$

Therefore, the expected repair and replacement cost rate is

$$C_F(N,K;T) = \frac{c_R + c_m\int_0^{NT}\overline{P}_K(t)h(t)\mathrm{d}t}{\int_0^{NT}\overline{P}_K(t)\mathrm{d}t}, \tag{3}$$

where c_R = replacement cost at time NT or at repair K, and c_m = cost of each minimal repair.

The integrated model in (3) includes the following replacement policies: When the unit is replaced at time $T(0 < T < \infty)$,

$$C(T) \equiv \lim_{K\to\infty} C_F(1,K;T) = \frac{c_R + c_mH(T)}{T}, \tag{4}$$

which agrees with the expected cost rate of the original periodic model with time T [25,26]. When the unit is replaced at time NT $(N = 1, 2, \cdots; 0 < T < \infty)$,

$$C(N;T) \equiv \lim_{K\to\infty} C_F(N,K;T) = \frac{c_R + c_mH(NT)}{NT} \quad (N = 1, 2, \cdots). \tag{5}$$

When the unit is replaced at repair K $(K = 1, 2, \cdots)$,

$$C(K) \equiv \lim_{N\to\infty} C_F(N,K;T) = \frac{c_R + c_mK}{\int_0^\infty \overline{P}_K(t)\mathrm{d}t} \quad (K = 1, 2\cdots). \tag{6}$$

When the unit is replaced at repair K $(K = 1, 2, \cdots)$ or at time $T(0 < T < \infty)$, whichever occurs first,

$$C_F(K;T) \equiv C_F(1,K;T) = \frac{c_R + c_m\int_0^T\overline{P}_K(t)h(t)\mathrm{d}t}{\int_0^T\overline{P}_K(t)\mathrm{d}t}. \tag{7}$$

3.2. Optimum policies

When the failure rate $h(t) \equiv \mathrm{d}H(t)/\mathrm{d}t$ increases strictly with t to $h(\infty) = \infty$, we find optimum replacement policies of *T, N* and *K* for the above expected cost rates and comparisons among them are made in terms of cost rates.

3.2.1. Optimum T*, N* and K*

We find optimum T^*, N^* and K^* to minimize $C(T)$ in (4), $C(N; T)$ in (5) and $C(K)$ in (6), respectively.

(1) Optimum T*

Optimum T^* $(0 < T^* < \infty)$ to minimize $C(T)$ satisfies [25,26]

$$Th(T) - H(T) = \frac{c_R}{c_m}, \tag{8}$$

and the resulting cost rate is

$$C(T^*) = c_mh(T^*). \tag{9}$$