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Aggregate discounted warranty cost forecasting considering the failed-but-not-reported events

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ABSTRACT

Warranty claims usually incur substantial costs to the manufacturers. In practice, it is not accurate or economical to reserve money for future claims based on the prediction of the overall (life cycle) warranty cost. As a more appropriate alternative, the aggregate warranty cost forecast technique can reduce the liquidity risks and improve the warranty service efficiency. In addition, when a product fails during the warranty period, a warranty claim will be counted only when the customer reports the failure to the manufacturer. This paper focuses on forecasting the discounted warranty cost, which depends on the product sales and failure processes, warranty terms, and customer behaviors, over arbitrary time interval. To characterize the failed-but-not-reported phenomenon, a flexible time-dependent function is proposed. We derive the mathematical formulations of related factors to discuss the modeling process of total discounted warranty cost over an arbitrary time interval (TDWCATI). The impacts of warranty length and customer reporting behavior are explored. The expectation and variance of the TDWCATI are obtained under the pro-rata warranty policy and the nonrenewable minimal-repair policy, which shows that the TDWCATI is useful in planning future warranty services and budgets over a specific time period.

1. Introduction

Warranty services are provided for most commercial products by the manufacturers as a signal of product quality and reliability. Product repairs during the warranty period are often free or pro-rata of charge, which brings substantial costs to the manufacturer for managing and serving warranty claims. Typically, warranty cost accounts for 2–15% of product net sales [1]. To ensure the quality of warranty service and reduce the liquidity risk, manufacturers often need to set up an appropriate amount of money for future warranty obligations. It is very costly to deplete the warranty reserve, because the manufacturers have to look for emergent funding with a high interest rate to support the unpredicted claims. On the other hand, excess warranty reserve is not desired as well due to its opportunity cost [2]. Consequently, the warranty reserve is set based on the estimated future warranty expense. Therefore, a reasonable estimation of warranty cost is of great importance to the manufacturers.

The development of cost estimation for warranty reserve preparation can be dated back to 1960s when Menke [3] first quantified the total warranty cost for an anticipated production plan under a pro-rata warranty policy. The cost was simply evaluated as (warranty cost per unit)×(product lot size). Because of the time value of money and inflation, Amato and Anderson [4] extended Menke's work by considering the present value and incorporating corrections for price level changes. Their work provides a more reasonable cost basis for the determination of warranty reserve for non-repairable product when discounting and inflation rate are not negligible. Mamer [5] further examined discounted per unit cost under three different types of product warranty. Chukova and Hayakawa [6] proposed a model to estimate the expected warranty cost, in which the repair time for warranty claims cannot be ignored. The aforementioned studies are concerned with expected warranty cost. However, for predictive and decision-making purposes, the expected values alone may not be adequate for establishing the reserve fund (which involves risk). To address this issue, Polatoglu and Sahin [7] studied the probability distributions of warranty cost that incorporate customer repurchase behavior for a failed unit, and Bai and Pham [8] derived the mean and variance of the discounted warranty cost for repairable systems with minimal repairs.

Most of the research on warranty cost modeling only emphasizes the product and manufacturer related factors such as design reliability [9], maintenance strategies [10,11], and warranty policies [12,13]. One important factor that is often overlooked is the failure reporting behavior of a customer. An implicit assumption in most of previous

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studies is that a failure will definitely result in a warranty claim, i.e., the customer will definitely report the failure. However, it is common that customers will not return failed units to manufacturers, which is called failed-but-not-reported (FBNR) phenomenon [14,15]. Various reasons can be found for this phenomenon. For example, when a cheap product (such as USB flash memory, laser pointer, kettle, etc.) fails, the owner may just throw it away rather than return it to the manufacturer for repair. Besides price, product life cycle also has a significant impact on the reporting behavior. Due to rapid-technological innovation, products with short life cycles such as portable audio players, cellphones, and cameras are easily obsolete. Impelled by new designs and functions, customers tend to buy updated products and will not make warranty claims even when the failed products are still under warranty. In addition, warranty related factors can also influence the reporting behavior. Those factors include the convenience of access to the warranty service, the efficiency, and effectiveness of warranty service, etc. For instance, if it takes a long time to return the unit to the center or wait for the repairing process, a customer may ask a third-party service provider to repair the unit or buy a new one. Moreover, some of the important consumer related factors that affect the reporting behavior are lifestyle, income, wealth, occupation, and personality. For wealthy people, busy professionals, or professionals who can get reimbursed from purchases, they may not bother to make warranty claims. Such FBNR phenomenon can lead to a great reduction in warranty claims and costs if the event proportion is high. Thus, the over-reserved money will be a waste in this case and cause opportunity cost. To the best of our knowledge, such practical issues have not been well studied in warranty modeling and analysis. Patankar and Mitra [16] proposed two time-dependent execution functions to characterize the customer reporting behavior and examined the effects of FBNR phenomenon on warranty cost per unit. Wu [14] further derived warranty cost with the consideration of both FBNR and non-failed-but-reported behaviors. Xie and Liao [17] further derived the expected aggregate warranty demand, which explicitly captured the FBNR events from a realistic perspective. Essentially, the warranty cost per unit depends on various variables including product reliability, warranty policy, and reporting behavior.

The literature related to warranty cost calculation is vast. Two typical elements, which have been intensively studied, are per unit cost and total cost for a single lot sale (simply by multiplying the cost per unit and the size of the lot sale). However, such methods of evaluating warranty cost over a certain period of time are only applicable when all units are sold at the same time. The life cycle cost can help manufacturers evaluate a warranty program and plan for the warranty reserve, but it is a great waste of money to reserve a large amount of money for the whole life cycle at the beginning of the sales period. In practice, the units are usually sold to customers intermittently, thus, dynamic sales processes should be taken into consideration in warranty cost estimation. For manufacturers, the sales volume may change drastically during the product life cycle, for which the accuracy of a long-term forecasting cannot be guaranteed. Therefore, a more reasonable and realistic way for warranty expense management is to predict the warranty cost progressively and to reserve the money for a given period of time (which is flexible for both short-term and long-term forecasts). This periodic planning is not only more economically effective but also more accurate, because the estimates of input variables can be updated timely with the cumulative warranty claims. Ulrich et al. [18] developed a model to calculate warranty expense incurred during a given future period under a pro-rata policy. He implicitly assumed that all failures would be reported as warranty claims, which may not be true because of the FBNR phenomenon. Another factor that is overlooked in his model is the time value of money, which is important in cost planning and reserving. Ja et al. [19] determined the mean and variance of total discounted warranty cost for the stochastic sales over the life cycle of the product. Xie and Ye [20] and Xie et al. [21] derived the associated statistics of the discounted aggregate cost within a given period of time. However, these models cannot account for any time horizon and the unnegligible customer reporting behavior, i.e., the FBNR events.

In this paper, our objective is to calculate the mean and variance of the total discounted warranty cost during any period of time. To fill the research gap, we explicitly consider the FBNR phenomenon (an important factor that directly influences the number of warranty claims) in our model and optimize the warranty reserve decision for an arbitrary time interval. We propose a continuous function to characterize the non-decreasing FBNR behavior and provide a detailed discussion about the flexibility of our formulation. Other factors including stochastic sales process, warranty policy, and discounting methods are also incorporated in modeling the discounted warranty cost. Specifically, a nonhomogeneous Poisson process is utilized to describe the sales process. Analysis of warranty cost is carried out under both pro-rata warranty policy and nonrenewable minimal-repair policy. A continuous discounting method is considered. We first derive the general expressions for the mean and variance of the discounted cost, then, for some special cases, we can also provide closed-form expressions for these statistics. Indeed, the knowledge of expected discounted cost is useful for manufacturers to determine the size of a warranty reserve. However, the expected value alone cannot adequately inform the total cost and control the risks. As a good measurement of uncertainty, the variance could help managers to determine the risk of having insufficient or excess warranty reserve. Consequently, our approach can assist manufacturers to plan the warranty reserve over any finite time horizon. The contribution of our study is that, when the FBNR events are considered, we are able to forecast the discounted warranty cost as well as obtain the optimal policy for warranty reserve within an arbitrary time period, which advances the knowledge in the areas of warranty cost analysis and reserve management.

The rest of this paper is organized as follows. In Section 2, we formulate the associated factors involved in the cost modeling. We then develop a model for the discounted warranty cost and derive the corresponding mean and variance of the cost in Section 3. In addition, we provide explicit forms of the cost for some special cases. Section 4 presents some numerical results and discusses the managerial implications through simulation. Finally, Section 5 concludes the paper.

2. Model formulation

Let $TDC(t_s, t_e)$ denote the total discounted warranty cost over a given time interval $(t_s, t_e]$ where $t_s \le t_e$. It is obtained by discounting costs incurred within $(t_s, t_e]$ back to the beginning of the study t_s and then adding up all discounted values. Note that the warranty expense only arises for items who are still under warranty and whose failures are reported during the study period $(t_s, t_e]$. Essentially, $TDC(t_s, t_e)$ is a result of the following key determinants, such as sales process, product reliability, warranty policy, customer reporting behavior, and the discounting method.

Suppose that a manufacturer starts selling a product at time zero and sales end at time L, i.e., the product life cycle is L. The sales are assumed to occur according to a nonhomogeneous Poisson process $\{N(t); t > 0\}$ with an intensity function $\lambda(t)$. Define $N(\mu_1, \mu_2) = N(\mu_2) - N(\mu_1)$ and $\Lambda(\mu_1, \mu_2) = \int_{\mu_1}^{\mu_2} \lambda(t)dt$ as the cumulative number of units sold and the expected number of sales over the time interval $(\mu_1, \mu_2]$, respectively. Then, it is easy to know that the purchase time T of a unit sold during $(\mu_1, \mu_2]$ is a random variable with density.

$$p(t) = \frac{\lambda(t)}{\Lambda(\mu_1, \mu_2)}. \tag{1}$$

Let $f(x)$ be the probability density function of the product's lifetime X. To describe the FBNR events, we assume that when a unit fails at age x, it will be reported to the manufacturer as a warranty claim with a probability $1 - q(x)$. Thus, the FBNR probability $q(x)$ is linked to the