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Optimal loading of series parallel systems with arbitrary element time-to-failure and time-to-repair distributions

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ABSTRACT

Many real-life systems have series parallel structures, where some components or subsystems work in parallel while some others have to work in series or consecutively. This paper models dynamic performance of multi-state series parallel systems with repairable elements that can function at different load levels. Performance (productivity) and time-to-failure distribution of an operating system element depend on its load level. An element, upon failure, can be repaired with repair time obeying a known distribution. The entire system must satisfy a random demand during a fixed mission time or must complete a fixed amount of work. A discrete numerical algorithm is first proposed for evaluating instantaneous availability of a system element with a particular load level, which further defines stochastic process of the element's performance. A universal generating function technique is then used for assessing system performance metrics including expected system performance, expected probability of meeting system demand, expected amount of unsupplied demand over a particular mission time and expected time needed to perform a given amount of work for the considered system. The proposed methodology is applicable to arbitrary types of failure time and repair time distributions. Another original contribution of this work is formulating and solving elements loading optimization problems, which choose elements load levels to achieve one of the following objectives: maximum system expected performance, maximum expected probability of meeting demand during a time horizon, minimum total unsupplied demand during the mission, or minimum completion time for a given amount of work. As demonstrated through a case study of a power station coal transportation system, optimization results can provide effective guidance on optimal operational load of multi-state series parallel system elements.

1. Introduction

Series parallel systems containing a combination of series and parallel segments or subsystems are among intensively-studied structures in the system reliability research. Most of existing studies focus on special classes of the series parallel systems, where a number of purely parallel subsystems are connected in series or a number of purely series subsystems are connected in parallel [1,2]. Optimization problems such as redundancy allocation [3–6], joint reliability-redundancy allocation [7,8], selective maintenance policy [9,10], and joint redundancy-maintenance optimization [11,12] have been formulated and solved for these systems.

This paper models a general series parallel system containing an arbitrary combination of series and parallel structures of system elements. The system elements are repairable with repair time obeying known distributions. A general repair model based on the virtual/effective age concept is considered [13,14]. Particularly, three different levels including minimal repair, perfect repair, and imperfect repair are covered [15,16]. Under the minimal repair model, the failed element is restored to an "as bad as old" condition (same as that immediately before the failure); the virtual age of the repaired element is the same as that before the repair. Under the perfect repair model, the failed element is restored to an "as good as new" condition (for example through replacement by a brand new element); the virtual age of the element after the repair is simply reduced to 0. Under the imperfect repair model, the failed element can be restored to any condition between the former two cases; the virtual age of the repaired element is decreased by a certain amount dependent on repair type or efficiency.

Abbreviations: *cdf*, cumulative distribution function; *pdf*, probability density function; *pmf*, probability mass function; UGF, universal generating function (u-function); MSS, multi-state system; DSCTP, discrete-state continuous-time process

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Nomenclature

W	total amount of work in the mission
J_n	maximal possible number of failures of element n during the mission
τ	time horizon
T	mission time
N	number of elements composing the system
W	amount of work in the mission
Ω	random demand
$\boldsymbol{L}$	vector of loads of system elements $\boldsymbol{L}$=(l(1),…,l(N))
$A(\boldsymbol{L})$	expected probability of meeting demand during mission time for elements loading $\boldsymbol{L}$
$a(\boldsymbol{L},t)$	expected probability of meeting demand at time t for elements loading $\boldsymbol{L}$
$\Theta(\boldsymbol{L})$	expected total of unsupplied demand during the mission time for elements loading $\boldsymbol{L}$
$\theta(\boldsymbol{L},t)$	expected unsupplied demand at time t for elements loading $\boldsymbol{L}$
$E(\boldsymbol{L},W)$	expected mission time needed to complete amount of work W for elements loading $\boldsymbol{L}$
$S(\boldsymbol{L})$	expected MSS performance during the mission for elements loading $\boldsymbol{L}$
$s(\boldsymbol{L},t)$	expected MSS performance at time t for elements loading $\boldsymbol{L}$
$\langle T_j,X_j\rangle$	event when the j-th failure of an element occurs at time T_j and the element spends time X_j in operation mode before the failure
$Q_j(t,x)$	function representing joint distribution of random values T_j and X_j for element n
$a_{n,l}(t)$, $r_{n,l}(t)$	instantaneous availability and unavailability of element n with load level l
D_n	random repair time for element n
$d_n^{\min}$,$d_n^{\max}$	minimal and maximal possible realizations of D_n
$g_{n,l}$	performance of element n working with load level l
$f_{n,l}(t)$, $F_{n,l}(t)$	*pdf, cdf* of time-to-failure distribution of element n working with load level l
$\psi_n(t)$, $\Psi_n(t)$	*pdf, cdf* of repair time for element n
m	number of discrete intervals considered in the numerical algorithm
Δ	duration of a discrete time interval: $\Delta=\tau/m$
$\lfloor x \rfloor$	floor operation that returns the maximal integer not exceeding x
η_{nl}, β_{nl}	scale, shape parameters of Weibull time-to-failure distribution of element n for load level l
$c_{n,j}(t)$	probability that element n is under repair after j-th failure at time t
π_n	repair efficiency coefficient of element n
$G_{n,l}(t)$	random performance of element n working with load level l at time t
$V(\boldsymbol{L},t)$	random performance of MSS with elements loading $\boldsymbol{L}$ at time t
$w_k(\boldsymbol{L},t)$	probability that $V(\boldsymbol{L},t)=v_k$ for elements loading $\boldsymbol{L}$
$\varphi^{op}_{n,l}$, $\varphi^{rep}_{n,l}$	per time unit operation and repair costs for element n working with load level l
$\Phi^{op}_{n,l}$, $\Phi^{rep}_{n,l}$	expected operation and repair costs during time τ for element n working with load level l
$\Phi^{inv}_{n,l}$	investment cost for element n working with load level l
$\Phi^{ud}(\boldsymbol{L})$	expected cost of unsupplied demand during time τ for given elements' loading $\boldsymbol{L}$

Each element can function at different load levels. It has been shown by empirical studies that the workload applied to a system element when operating can affect its performance (productivity) and time-to-failure distribution [17–19]. Consequently, the overall system performance can heavily depend on the selected element load levels. Thus optimal element loading problems become relevant and important to address for the design and operation of systems supporting different levels of element workloads.

To solve the optimal element loading problem for the considered series parallel system, a discrete numerical algorithm is first proposed to assess instantaneous availability of a system element functioning at a particular load level. A universal generating function (UGF) based reliability block diagram method is then applied to determine the system performance in terms of a discrete-state continuous-time process (DSCTP). Several performance metrics are evaluated, including expected system performance (amount of work completed), expected probability of meeting demand and expected amount of unsupplied demand over a particular mission time, as well as expected time for performing a specified amount of work.

Note that previous studies like [20–24] also adopted the DSCTP model and UGF technique to determine system dynamic behavior. These works however did not address element loading, and assumed that time-to-failure of system elements follows the exponential distribution with the memoryless property, allowing to use the Markov process model for system performance evaluation. In [25] the optimal loading problem was considered for stationary availabilities of elements in series-parallel systems, assuming exponential time-to-failure and time-to-repair distributions. In [26] the optimal loading problem was solved for elements with arbitrary time-to-failure and time-to-repair distributions. However the work of [26] is only applicable to single-element systems and the instantaneous availability of the element was not addressed, which did not allow the dynamic behavior of complex series parallel systems containing multiple elements to be analyzed.

In this work we make extensions to series parallel systems consisting of elements with different load levels and arbitrary time-to-failure and time-to-repair distributions. Based on the procedure proposed for evaluating element instantaneous availability and dynamic system performance, we formulate and solve the optimal loading problems, which choose elements load levels to maximize system expected performance, maximize expected probability of meeting demand during a time horizon, minimize total unsupplied demand during the mission, or minimize completion time for a given amount of work.

Monte Carlo simulations are a powerful tool that has been used for analyzing various types of complex systems [27–29]. They can typically offer great generality in the representation of system behavior, and can be potentially used for analyzing the series parallel system considered in this work. However, the simulation method is often expensive in computational requirements. Based on the suggested element instantaneous availability evaluation, the UGF technique implemented in this work is a straightforward universal approach of recursively applying aggregation procedures for obtaining performance distribution of complex system structures. This approach proved to be effective for different types of systems [30]. As shown in the published work [31], it allows obtaining system performance behavior in a much shorter time than the Monte Carlo simulations, which is especially important for solving the optimization problems considered in the paper.

The organization of the rest of the paper is as follows. Section 2 presents the repairable series parallel system model, definitions of performance metrics and description of problems addressed. Section 3 presents the proposed algorithm for evaluating instantaneous availability of system elements. Section 4 presents the UGF-based technique for determining stochastic process of the system performance. In Section 5, a power station coal transportation system is analyzed and optimized to illustrate application of the proposed methodology and optimization problems. Lastly, conclusions and directions of future