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An importance measure for multistate systems with external factors

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ABSTRACT

Many technical systems are operated under the impact of external factors that may cause the systems to fail. For such systems, an interesting question is how those external factors and their impacts on the system can be identified at an earlier stage. Importance measures in reliability engineering are used to prioritise weak components (or states) of a system. Component failures and the impact of external factors in the real world may be statistically dependent as external factors may affect system performance. This paper proposes a new importance measure for analysing the impact of external factors on system performance. The measure can evaluate the degree of the impact of external factors on the system and can therefore help engineers to identify the factors with the strong impact on the system performance. A real-world case study is used to illustrate its applicability.

1. Introduction

Importance measures are widely used to identify the weakest component of a system and to support system improvement activities in reliability engineering. Kuo and Zhu [1–3] summarise the concepts of importance measures in reliability and their applications in a wide spectrum of different areas. These measures can also provide valuable information that facilitates the safety and efficient operation of systems at different phases. For example, identifying the weakness of a system and understanding how the failure of each individual component affects the reliability of the system are crucial at the design phase. Engineers may then allocate resources for important components during the system operation stage and maintain the reliability of a system at a certain level. Importance measures are also used at the system maintenance phase to help engineers minimise maintenance cost and prolong the life of the system.

In binary systems, Birnbaum [4] originally defines the component importance, which evaluates the effect of changing the reliability of a component on the reliability of the system. Since then, many importance measures of binary systems are proposed from different perspectives [5–9]. For example, Wu and Coolen [10] introduce a cost-based importance, which extends the well-known Birnbaum importance. Borgonovo et al. [11,12] propose differential importance measure and time-independent reliability importance measure.

Many real-world systems have multiple states, ranging from a perfectly functioning state to one of complete failure. Several studies have been conducted to evaluate the reliability and performance of multistate systems [13–16]. To explore multistate systems, authors frequently use importance measures to identify the most critical components that facilitate the improvement and prioritization of system performance. For instance, Griffith [17] formalises the concept of system performance through expected utility and studies the effect of component improvement on system performance by generalizing Birnbaum importance. Zio and Podofillini [18] generalise the measure of Birnbaum importance with the performance level of multistate systems in contrast to binary systems that utilise Monte Carlo simulation. Wu and Chan [19] define a new utility importance of components of multistate systems to measure the importance of states. Ramirez–Marquez and Coit [20,21] present composite importance measures to identify and rank multistate components based on their impact on the reliability behaviuor of multistate systems. Ramirez–Marquez et al. [22] propose a multistate redundancy importance measure that provides information on the potential of components for improvement. Levitin et al. [23] consider the commonly used importance measures in multistate systems. Peng et al. [24] study the component reliability and importance of criticality to systems with degrading components. Tyrväinen [25] presents new risk importance measures applicable to a dynamic reliability analysis approach with multi-state components. Si and Dui et al. [26–28] propose an integrated importance measure to evaluate the effects of transition of components on system performance.

Many technical systems operate under the impact of external factors, such as intentional attacks, accidents, environmental factors, or natural

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Acronyms

BN	Bayesian network
MDD	Multistate decision diagram
HUD	Head-up display

Notations

n	number of components
N	number of external factors
i	index of component i, $i = 1, 2, \dots, n$
X_i	state of component i, $X_i = 0, 1, 2, \dots, M_i$
k	index of external factor k, $k = 1, 2, \dots, N$
Y_k	state of external factor k, $Y_k = 0, 1, \dots, S_k$
a_j	performance level corresponding to state j of the system
U	expected performance of a system
X	$(X_1, X_2, \dots, X_n)$: state vector of the components
Y	$(Y_1, Y_2, \dots, Y_N)$: state vector of the external factors
$\Phi(X)$	system structure function, $\Phi(X) = \Phi(X_1, X_2, \dots, X_n)$
$(\cdot_i, X)$	$(X_1, \dots, X_{i-1}, \cdot, X_{i+1}, \dots, X_n)$
P_{im}	$\Pr\{X_i = m\}, m = 0, 1, 2, \dots, M_i$
ρ_{im}	$\rho_{im} = \Pr\{X_i \geq m\} = P_{im} + P_{i(m+1)} + \cdots + P_{iM_i}$
n_{Path}	number of the MDD paths
l	index of MDD paths l, $l = 1, 2, \dots, n_{Path}$
$Path_l$	MDD paths l
n_{dPath}	number of the divided MDD paths
l_d	index of divided MDD paths l_d, $l_d = 1, 2, \dots, n_{Path}$
$dPath_{l_d}$	divided MDD paths l_d

disasters, these factors may cause damage of system components [29]. External factors such as fires, storms, earthquakes, high and low temperatures are often considered in probabilistic safety assessment (PSA), which is a widely used risk assessment tool in many industries such as the nuclear power industry, in which abnormal events, or external factors, may affect the normal operation of the facility in a firm [30]. A well-known example of the impact of external factors on a technical system is the Fukushima Daiichi nuclear disaster in Japan, which was initiated primarily by the tsunami following the Tōhoku earthquake on 11 March 2011 and caused several hydrogen-air chemical explosions. Another example is: the reliability of water mains is affected by environmental factors such as soil properties and temperature. Existing importance measures [31–34] are mainly concerned about system performance that resulted from changes in component reliability in terms of random failures, common cause failures and human errors.

A vital problem in engineering is to identify the factors with the strong impact on the system performance. Importance measure that can evaluate the degree of the impact of external factors on the system should therefore be developed to help engineers to protect the system from damage and further to improve the performance of the system. However, existing relevant research mainly analyses the protection of external factors on the system and the optimal defence based on different algorithms. The research in this area includes, for example, Levitin et al [35–39] estimate the protection for the impact of external factors on the system's survivability based on the universal generating function method. Zhang and Ramirez-Marquez [40] develop optimal protection strategies for critical infrastructures against intentional attacks. Shin and Kim [41] analyse the flight envelope protection systems to prevent an aircraft from exceeding structure limits. Considering mutually exclusive events and common cause failures, Vaurio [42–46] develops importance measures and their applications in fault tree techniques, multi-phase missions and non-coherent systems for the reliability and risk analysis.

It can be seen from the above examples that measuring the importance of external factors and then identifying possible hazards are vitally important. In practice, component failures and the impact of external factors may be statistically dependent as external factors may affect system performance. This paper proposes a new importance measure to evaluate the impact of external factors on a system performance.

The rest of this study is organised as follows. Section 2 introduces the importance measure of external factors. Section 3 evaluates the system performance based on the importance. Section 4 provides the method for evaluating the importance measure of external factors. Section 5 uses a case study to illustrate the applicability of the proposed measure. Section 6 concludes the paper.

Assumptions

1) The state space of component i is $\{0, 1, 2, \dots, M_i\}$ and the state space of the system is $\{0, 1, 2, \dots, M\}$, where 0 represents the completely failed state of the system/components and M_i (M) is the perfectly functioning state of component i (system). The performance of component i (the system) deteriorates from M_i (M) to 0.
2) The state space of external factor k is $\{0, 1, \dots, S_k\}$, where 0 represents that the external factor can cause the complete failure of the system. S_k represents that the external factors has no impact on the system. The severity decreases from 0 to S_k.
3) All external factors (states) are statistically independent.
4) The states of component i is impacted by external factors. All external factors and their states are known.

2. Importance measure with external factors

External factors may have impacts on system/component reliability. The state of an external factor represents the impact severity of the external factor. For example, state 0 of the external factor represents that the external factor can cause the complete failure of the system. With the change of impact severity of external factor, the external factors may change from one state to another state, and cause the system partial failure or complete failure. For example, in a system of water mains, when the temperature rises to 65 °C, the pipe may fail.

We assume there are N external factors, which affect system performance. The change of an external factor from one state to another may affect the states of components in a multistate system. Therefore, using the total probability formula and Assumption 3), the probability of component i being at state m is given as:

$$\begin{aligned}
P_{im} &= \Pr(X_i = m) \\
&= \sum_{b_1=0}^{S_1} \sum_{b_2=0}^{S_2} \cdots \sum_{b_N=0}^{S_N} \big(\Pr(Y_1 = b_1, Y_2 = b_2, \dots, Y_N = b_N) \\
&\quad \Pr(X_i = m | Y_1 = b_1, Y_2 = b_2, \dots, Y_N = b_N)\big) \\
&= \sum_{b_1=0}^{S_1} \sum_{b_2=0}^{S_2} \cdots \sum_{b_N=0}^{S_N} \big(\Pr(Y_1 = b_1)\Pr(Y_2 = b_2) \dots \Pr(Y_N = b_N) \\
&\quad \Pr(X_i = m | Y_1 = b_1, Y_2 = b_2, \dots, Y_N = b_N)\big) \\
&= \sum_{b_k=0}^{S_k} \Pr(Y_k = b_k) \sum_{b_1=0}^{S_1} \cdots \sum_{b_{k-1}=0}^{S_{k-1}} \sum_{b_{k+1}=0}^{S_{k+1}} \cdots \sum_{b_N=0}^{S_N} \\
&\quad \times \left(\left(\prod_{r=1, r\neq k}^{N} \Pr(Y_r = b_r) \right) \Pr(X_i = m | Y_1 = b_1, \dots, Y_N = b_N) \right).
\end{aligned}$$

Denote

$$f_{Y_k=b_k}(X_i = m) = \sum_{b_1=0}^{S_1} \cdots \sum_{b_{k-1}=0}^{S_{k-1}} \sum_{b_{k+1}=0}^{S_{k+1}} \cdots \sum_{b_N=0}^{S_N} \left(\left(\prod_{r=1, r\neq k}^{N} \Pr(Y_r = b_r) \right) \times \Pr(X_i = m | Y_1 = b_1, \dots, Y_N = b_N) \right).$$

For convenience, let $b_k = b$. We then obtain

$$P_{im} = \Pr(X_i = m) = \sum_{b=0}^{S_k} \left(\Pr(Y_k = b) f_{Y_k=b}(X_i = m)\right), \tag{1}$$