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Principal component analysis and polynomial chaos expansion for time-variant reliability problems

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A B S T R A C T

Time-variant reliability analysis aims to evaluate the probability that an engineering system successfully performs its intended function throughout its service life. The model describing the system behavior should then include the time-variant uncertainties. This results in an even more computationally expensive model what makes the surrogate modeling a promising tool. While Kriging has been widely investigated in the literature, this work explores the Polynomial Chaos Expansion (PCE) in the time-variant reliability field. First, the time interval of study is discretized and an instantaneous performance function is associated to each time node. Afterwards, a principal component analysis is performed in order to represent all these functions with a reduced number of components that are then approximated using PCE. An adaptive algorithm is proposed to automatically enrich the experimental design until the target accuracy is reached. Finally, a global surrogate model of the time-variant response is obtained on which the Monte–Carlo simulation method can be easily applied. This allows to obtain the complete evolution in time of the probability of failure with a unique reliability analysis. Results show that the proposed method is suitable for high dimensional time-dependent problems considering non-stationary and non-Gaussian processes.

1. Introduction

Time-variant reliability analysis is concerned with the calculation of the probability that a mechanical system fulfills its intended function throughout its lifetime. A realistic analysis needs to take into account both the random and temporal character of material properties, geometry parameters and loads. The time dependency adds an extra complexity to the classical reliability problem by introducing the time dimension. However, such an analysis is of vital importance for reducing the lifecycle cost [1], improving the sustainability of the maintenance [2] and setting a schedule for preventive condition-based maintenance [3]. Research in the field of time-variant reliability analysis is still ongoing to develop methods that procure the best compromise between efficiency and accuracy. Even though other methods exist for time-dependent reliability problems, the most widely used methods can be generally grouped into two main categories: the outcrossing methods and the extreme-value methods.

The outcrossing approach estimates the probability of failure through the average of the crossing rate during the structure lifetime. An outcrossing occurs if the performance function passes from the safe zone to the failure zone. Numerous methods have been proposed in the literature [4–6] to calculate the crossing rate using the asymptotic integration approach. The most common methods [7,8] are based on the Rice's formula [9] that assumes all upcrossings are independent. Their accuracy is therefore poor for problems with low reliability levels. Improvements have been made by relaxing this assumption using joint upcrossing rates [10] and the first order sampling approach [11]. In general, these methods are based on the First Order Reliability Method (FORM) which is inaccurate for problems with nonlinear limit state functions and multimodal statistical properties. Based on the works of Hagen and Tvedt [12,13], the widely used PHI2 method was proposed [14,15] using a two-component parallel system analysis and FORM. Despite its high efficiency, PHI2 provide only an upper bound of the cumulative probability of failure and thus may grossly underestimate the true reliability level.

The extreme value approach takes interest in the global extreme response of the mechanical system with respect to time. Failure is thereby considered if the value of interest is smaller than a given threshold. In this case, time-*invariant* reliability tools can be used if the distribution of the extreme value is accurately obtained. The specific problem where the time-dependency is introduced by only one loading stochastic process is treated in [16]. In [17] a Nested Extreme Response Surface

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(NERS) method is proposed. It relies on a double-loop procedure: in the first loop, the global extremes of the response are obtained through the Efficient Global Optimization (EGO) [18] whereas in the second loop the Kriging method is employed to model the time at which the extreme response occurs. An improvement of the method is proposed in [19] where samples from random variables and time are drawn simultaneously. However, EGO-based methods are unaffordable for problems where time is involved through stochastic processes. In this case, the system response may have several peaks and the computational cost becomes important. In [20], a method allowing to consider multiple input stochastic processes is proposed. First, the processes are discretized using a spectral decomposition. Then, Kriging models are used to approximate the limit state function at each discretized time node using a confidence-based adaptive sampling technique. The extreme value of the response can thus be obtained by evaluating the obtained surrogates. It is worth noting that the discretization of stochastic processes considerably increases the dimension of the problem (i.e. the total number of input random variables) and consequently reduces the computational efficiency of Kriging [21]. To overcome this issue, the SILK method was recently proposed in [22,23]. It is based on a one dimensional classification approach and allows to represent each stochastic process with a single variable. However, being based on Kriging, SILK may lack efficiency when a large experimental design is needed and when the surrogate model needs to be evaluated at a high number of new trial points. This requires the inversion of the covariance matrix of the Kriging model, whose size may dramatically increase. The computational burden is even higher when stochastic processes are involved with small autocorrelation lengths.

The Polynomial Chaos Expansion (PCE) [24] is another popular surrogate modeling technique widely used in probabilistic analysis. Through this method, a response surface of the mechanical model is constructed using multidimensional orthogonal polynomials with respect to the distributions of the input random variables. Further developments have lead to the sparse version that can tackle high dimensional problems [25] and PCE has recently shown its efficacy in time-*invariant* structural reliability analysis [26]. However, its application for transient problems is still insufficiently explored in the literature [27].

This paper presents a new method called *t*-PCE, for time-variant reliability analysis using polynomial chaos expansion. A procedure is proposed to replace the complex time-dependent response by an easy-to-evaluate polynomial model. First, the time interval of interest is discretized into a finite number of time nodes, and an instantaneous performance function is associated to each node. In principle, each of these functions can be approximated with a distinct PCE. However in general, engineering systems are studied over a long time interval, hence the number of functions may be very high requiring an important number of expansions. To tackle this issue, a Principal Component Analysis (PCA) is performed to represent all the instantaneous functions with a reduced number of non-physical components. Afterwards, each component is approximated using a polynomial chaos surrogate before combining them in a global surrogate model. An adaptive algorithm is also proposed and allows to iteratively enrich the experimental design until the polynomial surrogates meet the required accuracy. The time-variant reliability analysis can be finally assessed easily using simulation-based methods, such as Monte–Carlo Simulation (MCS).

The *t*-PCE method is proposed for the general time-variant reliability problem where the model response is function of input random variables, stochastic processes and the time parameter. The efficiency of *t*-PCE is investigated for high dimensional problems involving non-stationary and non-Gaussian stochastic processes. Moreover, *t*-PCE provides at no extra computational cost, the evolution in time of the cumulative probability of failure over the structure lifetime. The rest of this paper is organized as follows. Section 2 recalls the basics of time-variant reliability analysis. In Section 3, first a background concept on polynomial chaos expansion is provided then the proposed method is presented. Two case studies of a cantilever tube structure and a two-dimensional truss structure are used to demonstrate the effectiveness of the proposed methodology in Section 4. Finally a concluding summary is given in Section 5.

2. Time-variant reliability analysis

Let us start by stating the problem of time-variant reliability analysis. Consider a mechanical system whose behavior is both random and time-dependent. Its performance function is then denoted by $G(\mathbf{X}, \mathbf{Y}(t), t)$, where $\mathbf{X}$ and $\mathbf{Y}(t)$ denote respectively the vectors of random variables and stochastic processes and t is the time parameter. $\mathbf{X}$ includes the random parameters that may be related to materials properties and geometry. $\mathbf{Y}(t)$ collects the time-dependent random loadings (e.g. wind, waves, traffic,…). The time parameter usually appears explicitly when the analytical evolution in time of the degradation phenomena are known (e.g. corrosion). The system reliability state is associated to the sign of $G(\mathbf{X}, \mathbf{Y}(t), t)$ and is defined as follows:

- failing state if $G(\mathbf{X}, \mathbf{Y}(t), t) < 0$;
- reliable state if $G(\mathbf{X}, \mathbf{Y}(t), t) > 0$;
- limit state if $G(\mathbf{X}, \mathbf{Y}(t), t) = 0$.

2.1. Discretization of stochastic processes

Stochastic processes are the mathematical tools used to represent time-dependent random loadings. Often stochastic processes are assumed to be Gaussian both for simplicity and due to the central limit theorem. Many works have then focused on developing methods for discretizing particularly Gaussian processes. Some methods use series expansion techniques, such as expansion optimal linear estimation (EOLE) [28], orthogonal series expansion (OSE) [29] and Karhunen-Loève (KL) expansion [30]. Reviews of these methods can be found in [31]. However in reality, loads may exhibit non-Gaussian properties and even non-stationary behavior over time [32]. Some methods are developed for simulating stationary non-Gaussian processes mainly through non-linear transformation of Gaussian processes [33]. In [34,35], a framework that uses KL is proposed for generating strongly non-Gaussian and non-stationary stochastic processes. It incorporates KL into an iterative mapping scheme to reproduce the non-Gaussian marginal distribution function. This method is retained in this work because it is both efficient and easy to implement.

Consider a stochastic process $Y(t)$ that is indexed on a bounded time domain $D_T = [t_0, t_f]$. It is completely described by its mean $\bar{y}(t)$, autocovariance function $C(t_i, t_j)$ and marginal distribution. The classical KL expansion consists in approximating $Y(t)$ as follows:

$$\boldsymbol{Y}(t) \approx \bar{y}(t) + \sum_{k=1}^{M} \sqrt{\gamma_k} \zeta_k \boldsymbol{f}_k(t) \tag{1}$$

where M is the truncation order of the KL expansion and ζ_k are uncorrelated random variables of zero mean and unit variance following the same probability density function of $Y(t)$. γ_k and $f_k(t)$ are respectively the eigenvalues and eigenfunctions of $C(t_i, t_j)$. They are solution of the homogenous Fredholm integral equation of the second kind given by:

$$\int_{D_T} \boldsymbol{C}(s,t) \boldsymbol{f}_k(s) d s = \gamma_k \boldsymbol{f}_k(t) \tag{2}$$

In some specific cases, Eq. (2) can be solved analytically however in general, numerical methods such as the Galerkin technique [36] are to be used. The algorithm iterates on $\{\zeta_k\}$ until finding the best set that ensures the convergence of the autocovariance and marginal distribution functions of the process toward the real ones.

2.2. Estimation of the probability of failure

When all the stochastic processes are discretized, the performance function can be expressed in terms of only random variables and deter-