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Stability and control of iterated non-linear transport solvers for fusion edge plasmas

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ABSTRACT

The application of numerical transport solvers for the steady state plasma boundary of magnetic fusion devices is related to the iterative approximation of a fixed-point of a non-linear map. Although 2D (axisymmetric) or even 3D transport solvers are routinely applied for the quantification of steady state plasma flows, unstable behavior can occur under certain conditions. A simple two-point model is applied to demonstrate the generic nature of this kind of unstable behavior which can occur when the fixed-point loses its stability and resulting in a period doubling route to chaos. Furthermore, it is demonstrated that wavelike oscillations can occur at low divertor temperatures. An adaptive relaxation scheme is presented which allows to suppress discrete and wavelike oscillations in order to stabilize the fixed-point iteration.

1. Introduction

The quantification of plasma flows in the domain near exposed surfaces is a key topic for the development of a magnetically confined fusion reactor. These flows can significantly affect the lifetime of plasma facing components, and hence, their control poses one of the major challenges for the successful operation of the next step fusion device ITER [1]. Computer simulations of the plasma boundary both guide the interpretation of present fusion experiments and aid the design activities for future devices. Toroidal symmetry is often assumed in computational models for tokamak configurations (the next step fusion device ITER is based on such a configuration), which allows to reduce the complexity of the numerical problem to two spatial dimensions. One such computational model is the B2-EIRENE code [2,3] (also referred to as SOLPS), which has meanwhile been established as a numerical tool for the performance analysis of the ITER divertor (see [4] and references therein).

However, despite the establishment of two dimensional models, three dimensional models have gained importance over the last years. This is because stellarator configurations (an alternative concept for the magnetic confinement) are intrinsically non-axisymmetric, and furthermore because 3D effects are relevant in tokamak configurations as well when non-axisymmetric resonant magnetic perturbations (RMPs) from external coils are applied. These are of considerable interest since the recent success in controlling edge localized instabilities (ELMs) by RMPs [5–8]. As ELMs are considered a major threat for the plasma facing components in ITER [9], ELM control by RMPs has been integrated in the latest ITER design [10]. A 3D computational model which allows to study the impact of RMPs in steady state ITER-like configurations is the EMC3–EIRENE code [11–13]: a coupled version of a Monte Carlo solver for fluid edge plasma transport (EMC3) and a Monte Carlo solver for kinetic transport of neutral gas (EIRENE).

Rather than calculating the evolution in time towards a stationary solution, the EMC3–EIRENE code provides a steady state solution by means of iterative approximation. The non-linear transport coefficients such as the heat conductivity are fixed during one application of the solver, and consequently, an iterative application is required for a self consistent solution. Put in mathematical terms, the transport solver can be represented by the non-linear operator $\Phi : X \mapsto Y$ with $X, Y \in \mathcal{P}$ which maps the plasma state X to the plasma state Y ($\mathcal{P}$ denotes the abstract set of all plasma states). A self consistent solution X^* of the underlying physical model requires that $X^* = \Phi(X^*)$, i.e. it requires that X^* is a fixed-point of Φ. The simulation procedure described above then formally represents a fixed-point iteration $X_{n+1} = \Phi(X_n)$ based on an (within certain limits arbitrary) initial plasma state X_0. However, only a very basic convergence assessment for 3D edge plasma simulations regarding the residual noise level intrinsic to Monte Carlo methods has been conducted so far [13,14].

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Recent simulations which explore access to so-called detached plasma states have shown unstable behavior, i.e. densities and temperatures do not converge to a steady level but begin to develop continuous oscillations. Periodic oscillations and chaotic behavior are indeed intrinsic features of iterated non-linear systems that can occur when the fixed-point loses its stability. Consequently, these numerical instabilities limit the applicability of the selected solver to the underlying physical problem. As the reliability of sophisticated numerical models for the boundary of fusion plasmas is dependent on stable convergence, we present an analysis of the simulation procedure itself. For this purpose, we introduce an iterated two-point model in Section 2, which allows to reduce the dimensionality (and hence the numerical complexity) of the problem while retaining characteristic physical and numerical features. Under-relaxation is a common approach to avoid divergence of the iterative solution, although there is no guarantee that a converged solution can be found [15] (this is in particular demonstrated by the second example in Section 2). However, a generalized form of under-relaxation known as stability transformation method (which includes special reflections in phase space) has been introduced [16,17], and it has been shown that a transformation exists that allows to stabilize a fixed-point. Nevertheless, the actual values of the transformation parameters remain unknown, and scanning through all elements of the transformation class set is unfeasible for extensive simulations.

Motivated by the analysis of the simulation procedure given in Section 2, we introduce an adaptive relaxation scheme in Section 3. The principal idea is to adjust the relaxation factor in a way that allows a fast and stable convergence. Introducing small perturbations to a system parameter in order to control chaos has been proposed some time ago [18], and in recent years some difficulties of this approach have been overcome by methods that tweak the relaxation factor for the purpose of accelerating the convergence [19]. E.g. for 2D recirculating flows, a convergence acceleration method has been presented [20] based on the ratio of the residual norms of the two momentum equations. Furthermore, a method for optimizing relaxation factors (SOAR) has been presented [21] which allows to improve the performance of the SIMPLE algorithm [15] for solving fluid flows. However, while the SOAR method allows to reduce the overall computation time by reducing the required number of iterations, it can increase the computation time per iteration by a factor of 4–40. Our method on the other hand, is based on a quick analysis of the history of characteristic parameters, and can be implemented into the EMC3–EIRENE code without essentially increasing the computation time per iteration. The concept is similar to the fuzzy control algorithm presented in [22] for a CFD solver in turbulent flow. But while this method is designed to keep the amplitudes of high frequency harmonics at small values, our method focuses on the oscillation cycle itself in order to allow convergence in the first place.

2. Two-point model analysis of iterated transport solvers

The well established generic two-point model (see Sections 5.2 and 5.4 in [23] for details) provides a very basic approach to characterize the plasma boundary. Its intention is to relate plasma conditions "upstream" (index u, i.e. at the last closed magnetic flux surface halfway between targets) to plasma conditions at the divertor targets (index t). It is based on fluid plasma transport equations which can be considerably simplified to:

$$2\, n_t\, T_t = f_{\mathrm{mom}}\, n_u\, T_u \tag{1}$$

$$T_u^{7/2} = T_t^{7/2} + \frac{7\, f_{\mathrm{cond}}\, q_\parallel\, L}{2\, \kappa_{0e}} \tag{2}$$

$$\left(1 - f_{\mathrm{power}}\right)\, q_\parallel = \gamma\, e\, n_t\, T_t\, c_{st}. \tag{3}$$

The upstream density $n_u\,[\mathrm{m}^{-3}]$ and the parallel component of the heat flux $q_\parallel\,[\mathrm{W\,m}^{-2}]$ are taken as (physical) control parameters, and $L\,[\mathrm{m}]$ is the upstream-to-target connection length. The sheath heat transmission coefficient $\gamma \approx 7$ and $\kappa_{0e} \approx 2000$ are constant parameters. Dependent variables are the upstream temperature $T_u\,[\mathrm{eV}]$, the downstream temperature $T_t\,[\mathrm{eV}]$ and the downstream density $n_t\,[\mathrm{m}^{-3}]$. The sound speed at the target is given by $c_{st}\,[\mathrm{m\,s}^{-1}] = \sqrt{2\,e\,T_t/m}$ and the ion mass m. A generic extension by correction factors $f_{\mathrm{power}}, f_{\mathrm{mom}}, f_{\mathrm{cond}} \in [0, 1]$ allows to include various processes which are neglected in the original basic two-point model (see below). An approximate solution can be derived for prescribed correction factors:

$$T_u \approx \underbrace{\left(\frac{7\, q_\parallel\, L}{2\, \kappa_{0e}}\right)^{2/7}}_{=T_{u0}} f_{\mathrm{cond}}^{2/7} \tag{4}$$

$$T_t = \underbrace{\frac{m}{2\, e}\, \frac{4\, q_\parallel^2}{\gamma^2\, e^2\, n_u^2\, T_{u0}^2}}_{=F_u}\, \frac{\left(1 - f_{\mathrm{power}}\right)^2}{f_{\mathrm{mom}}^2\, f_{\mathrm{cond}}^{4/7}} \tag{5}$$

$$n_t = \frac{f_{\mathrm{mom}}\, n_u\, T_u}{2\, T_t} = \frac{n_u\, T_{u0}}{2\, F_u}\, \frac{f_{\mathrm{mom}}^3\, f_{\mathrm{cond}}^{6/7}}{\left(1 - f_{\mathrm{power}}\right)^2}. \tag{6}$$

On the other hand, the following iterative scheme is introduced in order to mimic the energy balance solver Φ_T of an edge plasma transport code such as EMC3:

$$T_{t,(n+1)} = \frac{\left(1 - f_{\mathrm{power}}\right)\, q_\parallel}{\gamma\, e\, n_t\, c_{st}(T_{t,(n)})} \tag{7}$$
$$T_{u,(n+1)} = \frac{1}{T_{u,(n)}^{5/2}} \left[T_{t,(n)}^{5/2}\, T_{t,(n+1)} + \frac{7\, f_{\mathrm{cond}}\, q_\parallel\, L}{2\, \kappa_{0e}} \right]$$

which takes n_t as unmodified input. The combined particle and parallel momentum balance solver Φ_n of the EMC3 code can be represented by

$$n_{t,(n+1)} = \frac{f_{\mathrm{mom}}\, n_u\, T_u}{2\, T_t} \tag{8}$$

which takes T_u and T_t as unmodified input. A relaxation scheme $X_{n+1} \mapsto \alpha\, X_n + (1-\alpha)\, X_{n+1}$ (note that often $\alpha' = 1 - \alpha$ is referred to as relaxation factor) is applied to each of Φ_T and Φ_n in order to stabilize the iterative procedure, which has been found to be sufficient for most applications in the past. The operator Φ is then defined by the successive application of Φ_T and Φ_n.

Example 1. The fixed-point iteration for T_t is illustrated in Fig. 1 for the basic two-point model (that is $f_{\mathrm{power}} = 0, f_{\mathrm{cond}} = f_{\mathrm{mom}} = 1$). The control parameters are set to $n_u = 4.2 \cdot 10^{19}\,\mathrm{m}^{-3}$, $q_\parallel = \frac{5}{9} \cdot 10^8\,\mathrm{W\,m}^{-2}$ and $L = 50\,\mathrm{m}$. Initial conditions are set to $T_{u,(0)} = T_{t,(0)} = 40\,\mathrm{eV}$ and $n_{t,(0)} = 10^{19}\,\mathrm{m}^{-3}$. The approximate solutions from (4)–(6) are:

$$T_u^* \approx 81.35\,\mathrm{eV}, \qquad T_t^* \approx 8.76\,\mathrm{eV},$$
$$n_t^* \approx 1.95 \cdot 10^{20}\,\mathrm{m}^{-3}. \tag{9}$$

It can be seen that the iterative method converges only for relaxation factors $\alpha = 0.6$ and $\alpha = 0.8$, while oscillations occur for the smaller relaxation factors $\alpha = 0.2$ and $\alpha = 0.4$. Such oscillatory behavior is a feature of non-linear maps, the Logistic map being a famous example [24]. This map exhibits a "period doubling route to chaos", i.e. the oscillation period doubles at certain values of a control parameter, turning to chaotic behavior at some point.