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Redundancy optimization for series-parallel phased mission systems exposed to random shocks

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ABSTRACT

Systems performing consecutive non-overlapping mission phases in a random environment are considered. Each phase is performed by a specific 1-out-of-*n* subsystem consisting of statistically identical parallel elements with the same functionality. The environment is modeled by the Poisson process of shocks commonly affecting all elements, by increasing their failure rate. A method for evaluating the mission success probability for arbitrary redundancy level in each subsystem is presented. The constrained redundancy allocation problem is formulated and solved. Illustrative examples are presented.

1. Introduction

When redundancy is used for enhancing system reliability, the optimal redundancy allocation problem (RAP) arises and the number of redundant elements in different subsystems should be determined to provide the desired level of the entire system reliability (subject to constraints imposed on system cost, weight, size etc.). The RAP has been intensively investigated for systems with different structures and redundancy types [1,2]. Numerous existing papers are focused on active or hot standby redundancy systems (with 1-out-of-*n*: G or *k*-out-of-*n*: G series-parallel structures). The employed numerical methods include dynamic programming [3], integer programming [4,5], genetic algorithm (GA) [6,7], ant colony optimization algorithm [8], Tabu search [9], swarm optimization [10] etc. RAP has also been considered for the series-parallel multi-state systems with homogeneous [11–13] or heterogeneous [14–16] component choices for each subsystem. Refer to [17,18] for the state-of-the-art reviews on reliability optimization problems and the corresponding solutions.

When systems operate in a dangerous or hostile environment, they can be exposed to the external impacts/shocks effecting all system elements. The risks of these common cause failures can affect the optimal redundancy allocation, which has been taken into account in [19,20].

A system can undergo different consecutive stages/phases during the mission. Each phase is performed by a specific subsystem. There exists a vast literature on phased mission systems [21–24]. Reliability of the phased mission systems with internal/external common cause failures was analyzed in [25–30]. The RAP for such systems was considered in [31,32] and the optimal component grouping in [33,34]. However, in all the previous studies it was always assumed that the external impacts can cause elements failure with a given probability, but cannot cause elements deterioration and increase their failure rate.

In this paper we consider a situation when the phased mission system can experience numerous random shocks affecting all system elements and causing their deterioration. Each mission phase is characterized by a random shock process with the specific intensity. The phases are independent i.e. the phase duration and parameters of shock process do not depend on those of the previous phases. We present an algorithm for the mission success probability evaluation and solve the constrained RAP.

As a motivating example, consider a space mission that includes launching, travel, landing, surface drilling, chemical analysis of the obtained specimens, and data transmission to Earth. Each phase is performed by a specific subsystem. In each phase the entire system is exposed to specific environments, characterized by different shock rates (such as extreme temperatures, radiation bursts and electromagnetic surges affecting the electronic parts of subsystems). Depending on subsystem protection and nature of subsystem elements, the influence of shocks on the equipment degradation can be different for different subsystems. The shock rates for different mission phases are usually known from previous probes history or from physical analysis of different environments. To enhance the mission success probability, a certain level of redundancy can be provided in each subsystem. However stringer weight, space and cost constraints limit the redundancy level in each

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Acronyms

MSP	mission success probability
RAP	redundancy allocation problem
HPP	homogeneous Poisson process
NHPP	non-homogeneous Poisson process

Notation

N	number of phases (subsystems)
τ_n	duration of phase n
V_n	constant shock rate in phase n
$K(t)$	number of shocks in [0, t)
h_n	number of identical elements in subsystem n
S	MSP
C	system cost
C^*	cost constraint
s_n	probability that subsystem n completes its task
c_n	cost of element belonging to subsystem n
$\lambda_n(t)$	intrinsic failure rate of element belonging to subsystem n
Λ_n^o, Λ_n^i	intrinsic failure rate in operation, idle modes for element belonging to subsystem n
η_n	shock influence factor for element belonging to subsystem n
L_n	time to failure of element belonging to subsystem n
T_n	time till the end if the nth phase
$\varphi_n(T_n, k)$	$\Pr(L_n > T_n \vert K(t) = k)$
$\rho_n(T_n, k)$	$\Pr(L_n > T_n, K(T_n) = k)$
$\pi_n(T_n, k)$	$\Pr(K(T_n) = k)$
$P(\tau, m, V)$	probability that exactly m shocks happen during time τ in shock HPP with rate V

particular subsystem and in the entire system levels. Therefore, the optimal redundancy allocation problem arises.

Similar setting can be relevant for some military systems when the mission consists of consecutive phases (such as target detection, delivery, steering, detonation), consecutively performed by different subsystems. During the mission, the system acts in a hostile environment and is exposed to intentional attacks causing shocks affecting all subsystems simultaneously.

There exists a vast literature on redundancy allocation. However, to the best of our knowledge, the common effect of dynamic random environment on the redundancy allocation in phased mission systems have never been considered.

Combinations of internal failures and external shocks have been intensively discussed in the literature. See, e.g., references [35,36] for the case when these failure modes are independent and [37–39] for the dependence between the shock and wear processes. The latter was studied mainly in the framework of optimal preventive maintenance. In the model considered in our paper, shocks act directly on the internal failure rate increasing it on a certain value with each impact. This setting was not discussed in the literature for the RAP problem.

The paper is organized as follows. Section 2 formulates the redundancy allocation problem. Section 3 presents an algorithm for evaluating the mission success probability. Illustrative examples are presented in Section 4. Finally, concluding remarks are given in Section 5.

2. Problem formulation

A system is operating in a dynamic random environment modeled by a shock process $\{K(t), t \geq 0\}$, where $K(t)$ is a number of shocks in [0, t). The shock process is observed and its rate can be estimated from historical or simulated data. A system's mission presumes performing N consecutive non-overlapping independent phases. Each phase n ($1 \leq n \leq N$) has duration τ_n and is characterized by a specific shock rate V_n of the homogeneous Poisson process (HPP), thus forming a piecewise constant rate of the corresponding non-homogeneous Poisson process (NHPP) for all phases that will be denoted by $v(t)$. Each phase n is performed by specific subsystem consisting of h_n *statistically identical* elements in parallel, which for convenience, will be called 'subsystem n'. Each subsystem is switched off/unpowered after completing its specific mission phase. During the entire mission, all elements of a system are exposed to external shocks. The same shocks affect all system elements that are operating or have to operate at the next phases. Distinct from the conventional extreme shock models (see, e.g., [40] and references therein), shocks in this model do not cause immediate failure of the elements, but result in their gradual deterioration/ageing, which is expressed by increase in their failure rates. This is reflected in a failure model presented in the next section. The failures of elements under the same realization of the shock process are assumed to be *independent* events. The system succeeds to complete its mission if for any n ($1 \leq n \leq N$), at least one element of subsystem n survives until the end of phase n. The problem is to find the optimal system redundancy vector $\boldsymbol{h}=\{h_1,\ldots,h_N\}$ maximizing the mission success probability $S(\boldsymbol{h})$ subject to constraints on the number of elements in each subsystem and on the entire system cost (weight):

$$\max\ S(\boldsymbol{h}) = \prod_{n=1}^{N}\left(1-\left(1-s_n\left(\sum_{i=1}^{n}\tau_i\right)\right)^{h_n}\right)$$
$$\text{s.t. } 1 \leq h_n \leq H_n,\ C(\boldsymbol{h}) = \sum_{n=1}^{N} c_n h_n < C^*, \tag{1}$$

where $s_n(t)$ is the probability that the element of type n survives time t from the start of the mission, H_n is the maximum number of elements allowed for subsystem n, C^* is the maximum allowed system cost (weight).

In the general case, each phase can be performed by groups of subsystems composing more complex configurations. For example, during the travel phase of a space mission, a navigation and propulsion (trajectory correction) subsystems should operate simultaneously, which corresponds to a series-parallel configuration. This can be taken into account by modifying the mission success probability function (1). For example, in the case of two phases, the first of which is performed by the first N_1 subsystems and the second is performed by the rest N–N_1 subsystems, the mission success probability takes the form $S(\boldsymbol{h}) = (\prod_{n=1}^{N_1}(1-(1-s_n(\tau_1))^{h_n}))(\prod_{n=N_1+1}^{N}(1-(1-s_n(\tau_1+\tau_2))^{h_n}))$. The methodology presented in this paper can be applied for any configuration of groups of subsystems. However, for the sake of simplicity, we will further consider the simplest case (1).

3. Failure and survival model

The effect of shocks on elements belonging to subsystem n ($1 \leq n \leq N$) is described by the following stochastic model:

$$\tilde{\lambda}_n(t) = \lambda_n(t) + \eta_n K(t), \tag{2}$$

where $\lambda_n(t)$ is an intrinsic failure rate of identical elements in this subsystem (without shocks) and η_n is a shock influence factor describing the jump in the failure rate of these elements on occurrence of each shock [41]. The value of parameter η_n can be estimated by testing elements failure rates in different environments, characterized by specific known shock rates. Note that, $\{\tilde{\lambda}_n(t), t \geq 0\}$ is a stochastic process, which is often called in the literature a hazard (failure) rate process [42,43]. In each realization, the second term in the r.h.s. of (2) is increasing, which describes elements' deterioration/ageing. The conventional failure rate that corresponds to the process $\{\tilde{\lambda}_n(t), t \geq 0\}$ can be derived by obtaining the corresponding conditional (on survival) expectation. Specifically, for the HPP with rate V_n and the constant intrinsic failure rate Λ_n, it becomes the following increasing function [41]

$$\Lambda_n + \eta_n E[K(t) \vert L_n > t] = \Lambda_n + V_n(1-\exp\{-\eta_n t\}),\ 1 \leq n \leq N,$$

where L_n denotes the time to failure of an element from the subsystem n.